

MATH2300
Graph Theory Assignment 1

Due Monday October 4th at 5pm (Hand in at a lecture, tute, or to room 67-448)

1. A graph G is *self-complementary* if it is isomorphic to its complement G^c .
 - (a) Draw a self-complementary graph of order 5.
 - (b) Prove that a self-complementary graph must have order $p \equiv 0$ or $1 \pmod{4}$.
2. (a) A graph has 14 vertices and 25 edges and every vertex has degree 3 or 5. How many vertices of degree 3 does the graph have?
 - (b) A graph has 7 vertices, 10 edges, six vertices of degree a and one of degree b . What is b ?
3. Determine whether the following degree sequences are graphical; if a sequence is graphical, draw a graph having that degree sequence; if a sequence is not graphical, briefly explain why.
$$S_1 : 8, 5, 5, 4, 3, 2, 2, 2, 2.$$
$$S_2 : 7, 7, 4, 3, 3, 3, 2, 2, 1.$$
$$S_3 : 6, 6, 4, 3, 3, 3, 1.$$
4. Find a counterexample to each of the following statements:
 - (a) If G is a connected graph that contains only even vertices, then G contains no cut-vertices.
 - (b) If G is a connected graph such that every vertex of G lies on a cycle of G , then G contains no cut-vertices.
 - (c) If G is a connected graph with a cut-vertex, then G contains a bridge.
 - (d) If G is a connected graph with a bridge, then G contains a cut-vertex. (Yes, there is a counterexample to this one!)
5. (a) Determine $\lambda(K_{m,n})$ and $\kappa(K_{m,n})$, where $1 \leq m \leq n$.
 - (b) Determine $\lambda(K_{1,m,n})$ and $\kappa(K_{1,m,n})$, where $1 \leq m \leq n$.
 - (c) If G is a graph of order p (with $p \geq 2$), and if $\delta(G) \geq (p-1)/2$, prove that G is connected.
6. (a) Show that any graph G contains a path of length $\delta(G)$.
 - (b) Draw an eulerian graph with an even number of vertices and an odd number of edges, or prove that this is impossible.

End of Assignment