MATH2300
Graph Theory Assignment 2

Due Friday October 15$^{\text{th}}$ at 5pm (Hand in at a lecture, tute, or to room 67-448)

1. (a) Draw all (nonisomorphic) trees of order 5.

   (b) There are 125 unequal trees having vertex set $\{1, 2, 3, 4, 5\}$. These 125 trees can be partitioned into three classes such that all the trees in any given class are isomorhpic. How many trees are there in each of the three classes.

2. Find all trees T such that T^c is also a tree.

3. For the graph K_7, with vertex set $\{v_1, v_2, \dots, v_7\}$, find the depth-first search tree and the breadth first search tree.

4. A (p, q) graph is said to be *graceful* if it is possible to label the vrertices of G with distinct elements from the set $\{0, 1, \dots, q\}$ in such a way that when each edge uv is then labelled with the integer $|i - j|$, where i and j are the labels of u and v, every edge has a different label.

   (a) Prove that $K_{1,n}$ is graceful for all positive integers n.

   (b) Prove that every path is graceful.

   (c) Prove that every tree of order 5 is graceful.

5. Show that the graph $C_3 \times C_3$ is nonplanar.

6. Use the fact that the Petersen graph (which has 10 vertices and 15 edges) has girth 5 to prove that it is nonplanar.

End of Assignment