MATH2300
Graph Theory Assignment 3

Due Wednesday October 27$^{\text{th}}$ at 5pm (Hand in at a lecture, tute, or to room 67-448)

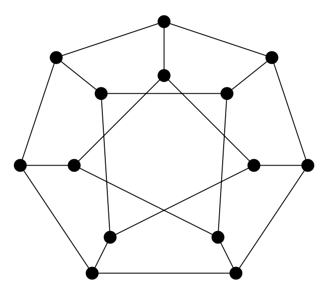
(Question 1 is worth 2%; Questions 2,3,4,5 are worth 1% each)

1. Find a minimum weight perfect matching in the weighted complete bipartite graph G with partite sets $\{v_1, v_2, \ldots, v_5\}$ and $\{u_1, u_2, \ldots, u_5\}$. The weight $w(v_iu_j)$ of the edge v_iu_j is given by $w(v_iu_j) = m_{ij}$ where $M = [m_{ij}]$ is the matrix shown below.

$$\begin{bmatrix} 3 & 4 & 8 & 9 & 10 \\ 7 & 5 & 6 & 7 & 4 \\ 8 & 12 & 5 & 9 & 6 \\ 4 & 5 & 9 & 7 & 8 \\ 7 & 10 & 13 & 6 & 6 \end{bmatrix}$$

2. Use the fact that the Petersen graph does not have C_{10} or C_4 as a subgraph to prove that it has no 1-factorization.

3. Find a 1-factorization of the graph shown below.



4. Determine the independence, clique, domination, and vertex chromatic number of the Petersen graph.

5. Show that if G is a graph with p vertices then $p \leq \chi(G)\beta(G)$.

BONUS QUESTION (worth an additional 2% to your final grade)

(a) Construct a 4-vertex chromatic triangle-free graph with 15 vertices.

(b) Suppose that the odd cycles in a graph G are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$.

End of Assignment