

## MATH2300 Linear Algebra half - Final Exam Revision

The following is an example of how part of Question 1 will read.

- Q1. (c?) For each of the following statements, STATE whether they are true or false. For those that are false, explain why and/or provide a counterexample. Do NOT give proofs if those statements which are true.
- (i) If  $\mathbf{0} \in \{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_p\}$ , then  $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_p$  are linearly dependent.
  - (ii) Every diagonal matrix is non-singular.
  - (iii) If  $A$  is an upper triangular square matrix then  $A$  is diagonalizable.
  - (iv) If  $A$  is invertible then  $\text{nullity}(A) = 0$ .
  - (v) If  $V = \text{span}(\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_n)$ , then each vector in  $V$  may be expressed as a unique linear combination of the vectors  $\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_n$ .
  - (vi) If  $S$  is a set of vectors and  $\mathbf{v} \notin \text{span}(S)$  then  $S \cup \{\mathbf{v}\}$  is a linearly independent set.
  - (vii) Let  $U, V$  and  $W$  be distinct vector spaces. If  $T_1 : U \rightarrow V$  and  $T_2 : V \rightarrow W$  are linear transformations then  $T_2 \circ T_1$  is a linear operator.
  - (viii) Any line in  $\mathbb{R}^2$  is a subspace of  $\mathbb{R}^2$  (under the standard addition and scalar multiplication).

In the exam there will be ten statements. Ideally, you will provide explicit counterexamples for those statements which are false. However, if your muse fails you on the day, or you are running out of time, then an explanation of how the statement might fall down will be sufficient (if it is correct!).

For example, (viii) above is false. An explicit counterexample would be the line  $x + y = 1$  (since  $(1,0)$  is on the line and  $(0,1)$  is on the line, but  $(1,1)$  is not on the line, and therefore this subset is not closed under addition). However, observing that "Any subspace of  $\mathbb{R}^2$  must contain the zero vector  $(0,0)$  and therefore any line not passing through the origin is not a subspace of  $\mathbb{R}^2$ " would also get you full marks.

You will also need to be able to do the following:

- Prove that a subset is a subspace of some larger vector space.
- Given a matrix  $A$  and its rref, find a basis for  $N(A)$ ,  $R(A)$ ,  $C(A)$ , and state rank and nullity of  $A$ .
- Prove that a transformation is linear.
- Find the matrix of transformation relative to some given basis of a linear transformation. Given this, find bases for the kernel and/or image of a linear transformation, and state rank and nullity of the transformation. (You will be given the rref of the matrix of transformation if required).

- (Orthogonally) Diagonalize a (symmetric) matrix. Note that if you have to orthogonally diagonalize a matrix, you will not need the Gramm-Schmidt process: each of the eigenspaces will have dimension 1, and so you will only need to normalise the basis vectors. (Remember that vectors from distinct eigenspaces of a symmetric matrix are already orthogonal!)
- Solve a system of linear first-order differential equations by using an appropriate change of variables. There will be two equations in two unknowns.

There will be several definitions tested.

Swotvac tutorials will be scheduled later in semester.