MATH2300
Graph Theory Problem Sheet 1

1. Draw a graph with five vertices having degrees 1,2,2,3, and 4.

2. Barney has invited five friends to a party at his house. After they all arrive, he asks them how many people (other than Barney himself) at the party they know. Each of the five gives a different answer. Is this possible? Explain your answer. (Assume that if person A knows person B, then person B knows person A.)

3. Suppose Fred and his wife attend a party with three other married couples. Several handshakes took place. No one shook hands with themself or with their spouse, and no one shook hands with the same person more than once. After all handshaking was completed, Fred asked each person, including his wife, how many hands they had shaken. Each person gave a different answer. How may hands did Fred shake? How many hands did his wife shake?

4. Draw all eleven nonisomorhpic graphs with 4 vertices.

5. Draw all 4-regular graphs with 7 vertices.

6. Determine whether the following degree sequences are graphical; if a sequence is graphical, draw a graph having that degree sequence; if a sequence is not graphical, briefly explain why.

$$S_1 \quad : \quad 7, 4, 4, 4, 4, 2, 2, 1.$$
$$S_2 \quad : \quad 5, 5, 3, 3, 1, 1.$$
$$S_3 \quad : \quad 5, 5, 4, 4, 3, 3, 3, 1, 0, 0$$

7. Let G be a graph with 12 vertices and 14 edges. If all the vertices of G are of degree either 2 or 3 then how many vertices of degree 2 does G have?

8. Draw a graph with $k(G) = \delta(G) = \Delta(G) = 2$.

9. Give an example of a graph G containing a vertex v such that $G - v$ has five components.

End of Problem Sheet 1