MATH2300
Graph Theory Problem Sheet 2

1. Let F be a forest with p vertices, q edges and k components. Show that $p = q+k$.

2. Show that not all graphs with p vertices and $p-1$ edges are trees.

3. Suppose that T is a tree with p vertices all of which are of degree 1 or 3. Show that T has exactly $(p-2)/2$ vertices of degree 3.

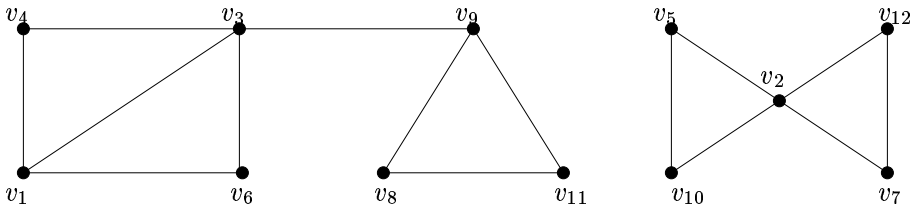
4. Let T be a tree with 21 vertices having degree set $\{1, 3, 5, 6\}$. If T has 15 vertices of degree 1 and one vertex of degree 6, how many vertices of degree 5 does T have?

5. Prove that if $d_1, d_2, \ldots, d_p$ is the degree sequence of a tree, then $d_1+1, d_2, d_3, \ldots, d_p, 1$ is the degree sequence of a tree.

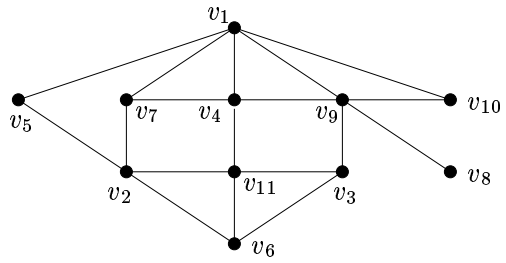
6. a connected graph G has degree sequence $8, 8, 7, 7, 6, 6, 6, 5, 4, 4, 3$. How many edges must be removed from G so that the resulting graph is a spanning tree of G?

7. For the graphs shown below, find the depth-first search forest and the breadth first search forest.

(a)



(b)



8. Determine all graphs G of order $p \geq 4$ such that the subgraph induced by *every* three vertices of G is a tree — or show that no such graphs G exist.

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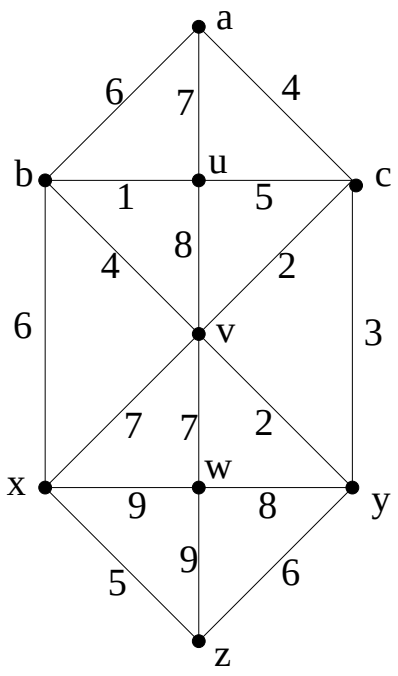
9. Use Kruskal's algorithm to find a minimum spanning tree in the weighted graph shown below.



10. Repeat the previous question using Prim's algorithm.

End of Problem Sheet 2