# MATH2300
Graph Theory Problem Sheet 3

1. (a) Show that for every positive integer n, the graph $K_{1,1,n}$ is planar. How many regions result when this graph is embedded in the plane?

   (b) Show that the graph $K_{2,n}$ is planar for every positive integer n.

2. Show that the graph obtained when an edge is removed from K_5 is maximal planar.

3. For $d = 2, 3, 4$ and 5, show that there is a d-regular maximal planar graph.

4. (a) Draw a nonplanar graph with p vertices and $3p - 6$ edges.

   (b) For what values of p does such a graph exist?

   (c) Prove that every disconnected graph with $p \geq 3$ vertices and $q = 3p - 6$ edges must be nonplanar.

5. Use Kuratowski's theorem to prove that the Petersen graph is nonplanar.

6. Are the following graphs planar? For each one, either draw it in a planar form, or else explain briefly why it is not planar.
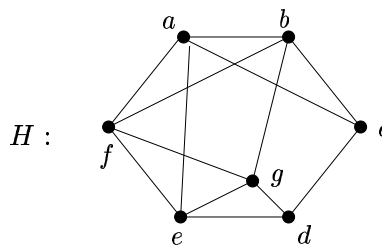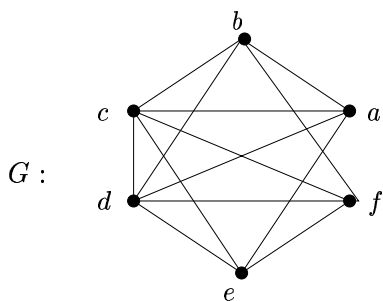
7. (a) Show that if a shortest cycle in a planar (p, q) graph has length k, then $q \leq k(p - 2)/(k - 2)$.

   (b) Hence (or otherwise) show that a planar bipartite graph of order $p \geq 3$ has at most $2p - 4$ edges.

End of Problem Sheet 3