Math 2400
Assignment 1
Due 11:50 a.m. on 26 March, 2014

Drop your paper into the white box with your tutorial number on the fourth floor of the Priestley building

1. (10 points) Consider the set $\mathbb{F} = \{0, 1, \alpha\}$. Define the operations $+$ and $\cdot$ on $\mathbb{F}$ so that $\mathbb{F}$, equipped with these operations, is a field. Verify the field axioms. Note: you may use rectangular tables to define the operations, as in Lecture 2.

2. (10 points) Suppose $\mathbb{F}$ is a field with finitely many elements. Prove that there exists a natural number n such that

$$\underbrace{1 + 1 + \cdots + 1}_{n \text{ times}} = 0.$$

3. (10 points) Prove that there exists no rational number x such that $x^2 = 6$. Hint: start by assuming that x exists and is equal to $\frac{p}{q}$, where p and q have no common divisors. Then use the fact that every natural number is either even or odd.

4. (5 points) Prove the inequality $||x| - |y|| \leq |x - y|$ for all $x, y \in \mathbb{R}$. This is sometimes called the second triangle inequality.

5. (10 points) Using the $\epsilon - N$ definition of the limit, prove that

$$\lim_{n \to \infty} \frac{1}{n^2 + n} = 0.$$

In other words, given $\epsilon > 0$, find explicitly a natural number N which satisfies the statement in the definition of the limit.

6. (5 points) Assume $(a_n)_{n=1}^{\infty}$ is a sequence of integers. Find a condition on the numbers a_n which would be necessary and sufficient for the convergence of the sequence. Don't forget to prove the necessity and the sufficiency.