Math 2400
Assignment 5
Due 11:50 a.m. on 6 June, 2014

Drop your paper into the white box with your tutorial number on the fourth floor of the Priestley building

Note: this assignment has SIX problems on TWO pages

1. (8 points) Find the derivative of the function $f : [1, \infty) \to \mathbb{R}$ defined by the formula
$$f(x) = \int_0^{x^4} e^{t^2}\, dt.$$

2. (8 points) Calculate the Taylor series for the function
$$f(x) = e^{x^3} + e^{2x^3}$$
around the point $a = 0$. Where does the series converge to the function $f(x)$?

3. (8 points) Prove that the series
$$\sum_{n=1}^{\infty} \frac{n^2 + \cos n}{e^{n^3}}$$
converges absolutely. (Hint: note that $|\cos n| \leq n^2$ for all $n \in \mathbb{N}$ and use the integral test.)

4. (8 points) Using Taylor series, calculate $\sinh 1$ correct to 6 decimal places. You must show your work. Simply finding $\sinh 1$ on a calculator will not earn you any credit.

5. Do the following series converge?

   (a) (5 points)
$$\sum_{n=3}^{\infty} \frac{1}{n(\log n)(\log(\log n))}$$
   (Hint: use the integral test and perform change of variable twice.)

(b) (5 points)

$$\sum_{n=3}^{\infty} \frac{1}{n + \log n}$$

(Hint: remember the inequality $\log n \leq n$.)

6. (8 points) Prove that the limit

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x^2yz}{x^8 + y^4 + z^2}$$

does not exist. (Hint: what if you approach the origin along the curve $z = x^4$, $y = x^2$?)