1. Calculate \( \lim_{n \to \infty} a_n \), \( \lim_{n \to \infty} \inf a_n \), \( \lim_{n \to \infty} \sup a_n \) for sequences

\[ \{a_n\}, \text{ where: } \lim_{n \to \infty} a_n \text{ may not exist.} \]

a) \( a_n = \cos \left( \frac{\pi n}{8} \right) \)

b) \( a_n = (1 + \frac{5}{n})^n \)

c) \( a_n = \frac{4+3n^2+n^2}{n^2+7} \)

2. Classify the following series as absolutely convergent, divergent, or conditionally convergent:

a) \( a_n = 2^n - n \)

b) \( a_n = \frac{1}{2^n + n} \)

c) \( a_n = \frac{n^n}{n!} \)

d) \( a_n = (-1)^n \frac{n^4}{n^4} \)

e) \( a_n = \frac{3^n + 1}{4^n + 5} \)

f) \( a_n = \left( -\frac{\ln n}{n} \right) \)

3. Given an \( \varepsilon-\delta \) proof for the continuity of the following functions at \( x = 2 \):

a) \( f(x) = x^2 + 7 \)

b) \( f(x) = \frac{1}{x+4} \)

4. Let \( I \) be an interval in \( \mathbb{R} \), let \( J \) be a subinterval (i.e., \( J \) is an interval, \( J \subseteq I \)). Let \( f \) be uniformly continuous on \( I \). Show that \( f \) is uniformly continuous on \( J \). Note: you need the following extension of the definition of continuity (one-sided continuity):

\[ f: \overline{(a,b)} \to \mathbb{R} \text{ is continuous from the right at } a \text{ if: given } \varepsilon > 0 \text{ there exists } \delta > 0 \text{ such that: } \]

\[ a < x < a + \delta \Rightarrow |f(x) - f(a)| < \varepsilon \]

Analogous for continuity (from the left) at \( b \) for \( f: (a,b] \to \mathbb{R} \). These definitions can be unified:

\[ f: I \to \mathbb{R} \text{ is continuous on } I \text{ if: given } \varepsilon > 0 \text{ there exists } \delta > 0 \text{ such that: } \]

\[ |x - x_0| < \delta, x_0 \in I \Rightarrow |f(x) - f(x_0)| < \varepsilon \]

b) Show that \( f(x) = x^2 \) is uniformly continuous on any interval of the form \((a, b)\), \( a < b \in \mathbb{R} \).

5. Is \( f(x) = \frac{1}{x} \) uniformly continuous on \((0, 1)\)?