MATH3401/3901: Complex Analysis/Advanced Complex Analysis

First Semester 2026
24/02/26

Assignment Number 1

Problem 1 *(3 points)* Graph the following regions in the complex plane:
a) $\{z\colon 2\mathrm{Re}\ z \le (\mathrm{Im}\ z)^2\}$;
b) $\{z\colon \pi/4 < \mathrm{Arg}\ z \le 3\pi/4\}$;
c) $\{z\colon |z-2i| \ge \pi\}$.

Problem 2 *(3 points)* Find *all* complex solutions of the following equations:
a) $\overline{z} = z$;
b) $\overline{z} + z = 0$;
c) $\overline{z} = 4/z$.

Problem 3 *(3 points)* Express the following in the form $x + iy$:
a) $\dfrac{i}{1-i} + \dfrac{1-i}{i}$; **b)** all the 3rd roots of $64i$; **c)** $\left(\dfrac{i+1}{\sqrt{2}}\right)^{1337}$.

Problem 4 *(5 points)* Recall the following (you do not have to prove it): For $n \in \mathbb{Z}_+$, $\sqrt{n} \in \mathbb{Q}$ if and only if n is a perfect square, i.e., $n = m^2$ for some $m \in \mathbb{Z}$.
Keeping this in mind, the set "$\mathbb{Q}$ adjoin $\sqrt{2}$" is defined by: $\mathbb{Q}(\sqrt{2}) = \{p+q\sqrt{2} : p, q \in \mathbb{Q}\}$.
a) Show that $\mathbb{Q}(\sqrt{2})$ is a *subfield* of $\mathbb{R}$ (i.e., it is a subset that is a field).
Hint 1: remember that $\mathbb{Q}$ and $\mathbb{R}$ are fields, and note that $\mathbb{Q}(\sqrt{2}) \subset \mathbb{R}$: hence you will get many, indeed, most of the properties almost free.
Hint 2: make sure you check closure.
b) Is $\sqrt{3} \in \mathbb{Q}(\sqrt{2})$? Explain your answer.

Problem 5 *(3 points)* A field $\mathbb{F}$ is called *ordered* if there exists a distinguished subset $\mathcal{P} \subset \mathbb{F}$, closed under addition and multiplication, such that the *trichotomy law* holds, i.e., For every $x \in \mathbb{F}$, precisely one of the following holds:
(i)$x \in \mathcal{P}$;
(ii)$x = 0$; or
(iii)$-x \in \mathcal{P}$.
(Note that we can then define an ordering via $x > y \Leftrightarrow x - y \in \mathcal{P}$).
Prove that $\mathbb{C}$ is not ordered. (Hint: first show that there must hold $-1 \notin \mathcal{P}$).

Problem 6 *(3 points)* Write one or at most two paragraphs on the *Schwarz-Christoffel transformation*. You should also include one relevant picture or diagram, appropriately referenced. Neither your paragraphs nor your picture should be from Wikipedia. Provide references as appropriate: see the UQ student guide at https://uq.pressbooks.pub/digital-essentials-write-cite-submit/chapter/module-overview/. Note that it is strongly recommended **not** to use CoPilot or similar generative AI for this question, or indeed for any question on the assignment. If you do so, you **must** reference your usage fully.

(Extra style points if the answer is in verse. Disclaimer: style points are not exchangeable for actual assignment points.)

Due: 10 A.M., Friday, 13/03/2026.
Current assignments will be available at `http://www.maths.uq.edu.au/courses/MATH3401/AssignmentsEtc.html`