MATH3401/3901: Complex Analysis/Advanced Complex Analysis

First Semester 2025
20/03/25

Assignment Number 2

Problem 1 *(2 points)* Determine the Möbius transformation (viewed as a mapping on $\overline{\mathbb{C}}$) mapping 2 to 0, i to ∞, and 0 to $-2i$.

Problem 2 *(4 points)* Let T be a mapping from Ω, a subset of $\mathbb{C}$, to $\mathbb{C}$. A *fixed point* of T is a point z satisfying $T(z) = z$.
a) Show: any Möbius transformation, apart from the identity, can have at most 2 fixed points in $\mathbb{C}$. (The identity is the transformation $z \mapsto z$).
b) Give examples of Möbius transformations having (i) 2; (ii) 1 and (iii) no fixed points in $\mathbb{C}$.

Problem 3 *(2 points)* For $z \in \mathbb{C}$, show:
a) $\sin \overline{z} = \overline{\sin z}$; **b)** $\cosh \overline{z} = \overline{\cosh z}$

Problem 4 *(3 points)* Find all solutions $z \in \mathbb{C}$ of the following (express your answers in the form $x + iy$):
a) $\log z = 4i$; **b)** $z^i = i$.

Problem 5 *(5 points)*
a) Prove that $\tanh^{-1} z = \dfrac{1}{2} \log \left(\dfrac{1+z}{1-z} \right)$, and note any restrictions on your domain.
b) Find all solutions $z \in \mathbb{C}$ of $\tanh z = i$ (express them in the form $x + iy$).

Problem 6 *(4 points)* Let Ω_1 and Ω_2 be nonempty, closed sets in $\mathbb{C}$.
a) Show that the set $\Omega_1 \cup \Omega_2$ is closed.
b) If instead Ω_2 is nonempty and open:
(i) could $\Omega_1 \cup \Omega_2$ still be closed?
(ii) Need it be closed?
Give proofs or examples/counterexamples.

Due: 10:00 A.M., Friday, 04/04/2025.

Current assignments will be available at
`http://www.maths.uq.edu.au/courses/MATH3401/AssignmentsEtc.html`