MATH3401/3901: Complex Analysis/Advanced Complex Analysis

First Semester 2026
27/03/2026

Assignment Number 3

Problem 1 *(6 points)* Show the following limits:

a) $\lim_{z\to\infty} \frac{4z^5}{z^5 - 142z} = 4$; **b)** $\lim_{z\to\infty} \frac{z^4}{z^2 + 142z} = \infty$;

c) $\lim_{z\to\infty} \frac{(az+b)^3}{(cz+d)^3} = \frac{a^3}{c^3}$ if $a, b, c, d \in \mathbb{C}, c \neq 0$.

Problem 2 *(3 points)* Show that the following functions are defined on all of $\mathbb{C}$, but are nowhere analytic (here $z = x + iy$):
a) $z \mapsto 2xy + i(x^2 + y^2)$ **b)** $z \mapsto e^y e^{ix}$.

Problem 3 *(4 points)* Determine where the function $z \mapsto x^3 + i(1-y)^3$ is:
a) analytic; **b)** differentiable (here $z = x + iy$).

Problem 4 *(3 points)* For $z = x + iy$, define $f(z) = \sqrt{|xy|}$.
a) Show that f satisfies the Cauchy-Riemann equations at the origin. (Note: you will need to use the definition of the partial derivative to calculate $u_x(0,0)$ and $u_y(0,0)$).
b) Show that f is not differentiable at the origin. (Hint: approach on a suitable line).
c) Explain why this doesn't contradict any of the results from class.

Problem 5 *(4 points)*
(a) Define precisely what it means for a curve in $\mathbb{C}$ to be *rectifiable.*
(b) Give an example of a non-rectifiable curve in $\mathbb{C}$. You don't have to prove it is non-rectifiable.
Note: you will need to provide at least one reference, properly cited. This is not allowed to be wikipedia.

Problem 6 *(2 bonus points)* Consider sequences defined as follows:

$$x_0 = 0, \quad x_1 = x_2 = 1, \quad x_3 = x_4 = 1 - \tfrac{1}{2}, \quad x_5 = x_6 = 1 - \tfrac{1}{2} + \tfrac{1}{4}, \cdots \quad \text{and}$$
$$y_0 = y_1 = 0, \quad y_2 = y_3 = 1, \quad y_4 = y_5 = 1 - \tfrac{1}{2}, \quad y_6 = y_7 = 1 - \tfrac{1}{2} + \tfrac{1}{4}, \cdots .$$

Set $z_n = x_n + iy_n$, and define I_n to be the closed line segment (i.e., including end-points) from z_n to z_{n+1}, for $n \in \mathbb{N}_0$. Finally, set:

$$\Omega = \bigcup_{n=0}^{\infty} I_n, \quad \text{and } \Lambda = \Omega \cup \{\tfrac{2}{3} + \tfrac{2}{3}i\}.$$

a) Show that Λ is connected. (A sketch is not a proof, but could be helpful in fixing ideas.)
b) Show that Λ is *not* piecewise affinely path connected, in the sense defined in class (Lecture 12).
c) Do these results contradict anything from class? Explain.

Due: 10:00AM, Friday, 01/05/2026

Current assignments will be available at
`http://www.maths.uq.edu.au/courses/MATH3401/AssignmentsEtc.html`