

Assignment Number 4

Problem 1 (2 points) Evaluate

$$\int_C \frac{\sin z}{(z+1)^7} dz,$$

where C is the circle of radius 4, centre 0, positively oriented.

Problem 2 (2 points)

Verify that the following functions u are harmonic, and in each case give a conjugate harmonic function v (i.e., v such that $u + iv$ is analytic).

- (a) $e^x \cos y$,
- (b) $x^2 - y^2 - 2y$.

Problem 3 (4 points)

- (a) Suppose that U solves a Neumann problem for Laplace's equation on a domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$. Show that $U + c$ also solves this problem for any $c \in \mathbb{R}$.
- (b) Does the same result hold for the corresponding Dirichlet problem?

Problem 4 (2 points)

Find a power-series expansion of the function $f(z) = \frac{1}{4-z}$ about the point $3i$, and calculate its radius of convergence.

Problem 5 (2 points)

Find a Laurent-series expansion of the function $f(z) = z^{-1} \sinh(z^{-1})$ about the point 0, and classify the singularity at 0.

Problem 6 (8 points) For each of the following functions, find and classify all singularities, including giving the order if the singularity is a pole. Also calculate the residue at each singularity.

- (a) $\frac{1 - \cos z}{z^2}$
- (b) $\frac{z^3}{(2+z)^3}$
- (c) $\frac{1}{(z^2+1)(z-1)^2}$
- (d) $\frac{e^z}{(1-z)^4}$

Due: 10:00AM, Friday, 23/05/2025

Current assignments will be available at

<http://www.maths.uq.edu.au/courses/MATH3401/AssignmentsEtc.html>