MATH3901: Advanced Complex Analysis

First Semester 2025
11/05/2025

Assignment Number 4

Problem 1 *(2 points)* Evaluate

$$\int_C \frac{\sin z}{(z+1)^7}\,dz,$$

where C is the circle of radius 4, centre 0, positively oriented.

Problem 2 *(2 points)*
Verify that the following functions u are harmonic, and in each case give a conjugate harmonic function v (i.e., v such that $u+iv$ is analytic).
(a) $e^x \cos y$,
(b) $x^2 - y^2 - 2y$.

Problem 3 *(4 points)*
(a) Suppose that U solves a Neumann problem for Laplace's equation on a domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$. Show that $U + c$ also solves this problem for any $c \in \mathbb{R}$.
(b) Does the same result hold for the corresponding Dirichlet problem?

Problem 4 *(2 points)*
Find a power-series expansion of the function $f(z) = \dfrac{1}{4-z}$ about the point $3i$, and calculate its radius of convergence.

Problem 5 *(2 points)*
Find a Laurent-series expansion of the function $f(z) = z^{-1}\sinh(z^{-1})$ about the point 0, and classify the singularity at 0.

Problem 6 *(4 points)* Define

$$f(z) = \begin{cases} z^5/|z^4| & z \neq 0 \\ 0 & z = 0. \end{cases}$$

(a) Show that f satisfies the Cauchy-Riemann equations at the origin. (Note: you will need to use the definition of the partial derivative to calculate the partial derivatives of u and v at the origin.)
(b) Show that f is not differentiable at the origin. (Hint: approach on lines).
(c) Explain why this doesn't contradict any of the results from class.

Problem 7 *(4 points)*
(a) Prove that the coefficients c_n in the expansion

$$\frac{1}{1-z-z^2} = \sum_{n=0}^{\infty} c_n z^n$$

satisfy the recurrence relation $c_0 = c_1 = 1$, $c_n = c_{n-1} + c_{n-2}$ for $n \geq 2$.
(b) What is the radius of convergence of the series in (a)?
(c) What would be a good name for the c_n's?

Due: 10:00AM, Friday, 23/05/2025

Current assignments will be available at
`http://www.maths.uq.edu.au/courses/MATH3401/AssignmentsEtc.html`