MATH3401: Complex Analysis

First Semester 2026
01/05/2026

Assignment Number 4

Problem 1 *(2 points)* Evaluate

$$\int_C \frac{\sin z}{(z+1)^7}\, dz,$$

where C is the circle of radius 4, centre 0, positively oriented.

Problem 2 *(2 points)*
Verify that the following functions u are harmonic, and in each case give a conjugate harmonic function v (i.e., v such that $u + iv$ is analytic).
(a) $e^x \cos y$,
(b) $x^2 - y^2 - 2y$.

Problem 3 *(4 points)*
(a) Suppose that U solves a Neumann problem for Laplace's equation on a domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$. Show that $U + c$ also solves this problem for any $c \in \mathbb{R}$.
(b) Does the same result hold for the corresponding Dirichlet problem?

Problem 4 *(2 points)*
Find a power-series expansion of the function $f(z) = \dfrac{1}{3-z}$ about the point $4i$, and calculate its radius of convergence.

Problem 5 *(2 points)*
Find a Laurent-series expansion of the function $f(z) = z^{-1}\sinh(z^{-1})$ about the point 0, and classify the singularity at 0.

Problem 6 *(8 points)* For each of the following functions, find and classify all singularities, including giving the order if the singularity is a pole. Also calculate the residue at each singularity.

(a) $\dfrac{1-\cos z}{z^2}$ (b) $\dfrac{z^3}{(2+z)^3}$ (c) $\dfrac{1}{(z^2+1)(z-1)^2}$ (d) $\dfrac{e^z}{(1-z)^4}$

Due: 10:00AM, Friday, 22/05/2026

Current assignments will be available at
`http://www.maths.uq.edu.au/courses/MATH3401/AssignmentsEtc.html`