

SCHOOL OF MATHEMATICS AND PHYSICS

MATH3401

Problem Worksheet

Semester 1, 2025, Week 11

- (1) Use the definition of limits of sequences, given in Lecture 30 page 3, to verify that the limit of the sequence

$$z_n = -2 + i \frac{(-1)^n}{n^2} \quad (n = 1, 2, \dots).$$

converges to -2 .

- (2) Find the Maclaurin series expansion of the function

$$f(z) = \frac{z}{z^4 + 9}$$

and calculate the radius of convergence.

- (3) Find the Taylor series of the function

$$f(z) = \frac{1}{1 - z}$$

about the point i and provide the radius of convergence.

- (4) Show that when $0 < |z| < 4$,

$$\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}.$$

(5) Write the two Laurent series in powers of z that represent the function

$$f(z) = \frac{1}{z(1+z^2)}$$

in certain domains, and specify those domains.

Hint 1: For one domain you should get

$$\sum_{n=0}^{\infty} (-1)^{n+1} z^{2n+1} + \frac{1}{z}.$$

For the other domain, you should get

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{z^{2n+1}}.$$

Hint 2: Observe that $(-1)^{n-1} = (-1)^{n-1}(-1)^2 = (-1)^{n+1}$.