SCHOOL OF MATHEMATICS AND PHYSICS

MATH3401
Problem Worksheet
Semester 1, 2025, Week 12

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(1) Use Cauchy's residue theorem (Lecture 33) to evaluate the integral of each of these functions around the circle $|z| = 3$ in the positive sense:

$$(a)\ \frac{\exp(-z)}{z^2}; \qquad (b)\ z^2 \exp\left(\frac{1}{z}\right); \qquad (c)\ \frac{z+1}{z^2 - 2z}.$$

(2) In each case, find the Laurent series of the function at its isolated singular point. Determine whether that point is a pole (determine its order), a removable singular point or an essential singularity. Finally, determine the corresponding residue.

$$(a)\ z \exp\left(\frac{1}{z}\right); \quad (b)\ \frac{z^2}{1+z}; \quad (c)\ \frac{\cos z}{z}; \quad (d)\ \frac{1-\cosh z}{z^3}; \quad (e)\ \frac{1}{(2-z)^3}.$$

Suggestion 1: Use the known series

$$\sum_{n=0}^{\infty} \frac{z^n}{n!}, \qquad \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}, \qquad \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}, \qquad (|z| < \infty).$$

Suggestion 2: For part (b) notice that $z^2 = (z+1)^2 - 2z - 1 = (z+1)^2 - 2(z+1) + 1$

(3) Find the value of the integral

$$\int_C \frac{3z^3 + 2}{(z-1)(z^2+9)} dz,$$

taken counterclockwise around the circle (a) $|z - 2| = 2$; (b) $|z| = 4$.

Ans. (a) πi*; (b)* $6\pi i$.