## SCHOOL OF MATHEMATICS AND PHYSICS

## MATH3401 Problem Worksheet Semester 1, 2025, Week 13

(1) Find the Laurent series expansion of

$$f(z) = (1 - z^2) \sin\left(\frac{1}{z}\right)$$

about the point z = 0, classify the singularity, and find the residue at that point.

(2) Use residues to evaluate the improper integral:

$$\int_0^\infty \frac{dx}{(x^2+1)^2}$$

Ans.  $\pi/4$ .

(3) Use residues to find the Cauchy principal value of the integral

$$\int_{-\infty}^{\infty} \frac{x \, dx}{(x^2 + 1)(x^2 + 2x + 2)}$$

Ans.  $-\pi/5$ .

- (4) Determine the number of zeros, counting multiplicities, of the polynomials
  - (a)  $z^4 + 3z^3 + 6$ ; (b)  $z^4 - 2z^3 + 9z^2 + z - 1$ ; (c)  $z^5 + 3z^3 + z^2 + 1$ inside the circle |z| = 2.
  - Ans. (a) 3; (b) 2; (c) 5.