SCHOOL OF MATHEMATICS AND PHYSICS

MATH3401/3901:
Problem Worksheet
Semester 1, 2025, Week 2

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(1) Prove that multiplication of complex numbers is commutative.

(2) Simplify each of these to a real number:

$$\text{(a) } \frac{1+2i}{3-4i}+\frac{2-i}{5i}; \qquad \text{(b) } \frac{5i}{(1-i)(2-i)(3-i)}.$$

(3) Use the properties of conjugates and moduli to show that

$$\text{(a) } \overline{\overline{z}+3i}=z-3i; \quad \text{(b) } \overline{iz}=-i\overline{z}; \quad \text{(c) } \left|(2\overline{z}+5)\left(\sqrt{2}-i\right)\right|=\sqrt{3}|2z+5|.$$

(4) Verify that $\sqrt{2}|z| \geq |\mathrm{Re}z| + |\mathrm{Im}z|$.

Suggestion: Reduce this inequality to $(|x|-|y|)^2 \geq 0$.

(5) Use de Moivre's formula to derive the following trigonometric identities:

$$\text{(a) } \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta; \qquad \text{(b) } \sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta.$$