MATH3401
Problem Worksheet
Semester 1, 2025, Week 4

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(1) Find all values of z such that

    (a) $e^z = -2$;

    (b) $e^z = 1 + \sqrt{3}$.

(2) Show that $\overline{\exp(iz)} = \exp(i\overline{z})$ if and only if $z = n\pi$, $(n = 0, \pm 1, \pm 2, \ldots)$.

(3) Show that

    (a) $\mathrm{Log}\,(1+i)^2 = 2\,\mathrm{Log}\,(1+i)$;

    (b) $\mathrm{Log}\,(-1+i)^2 \neq 2\,\mathrm{Log}\,(-1+i)$.

(4) Show that

    (a) the set of values of $\log\left(i^{1/2}\right)$ is

$$\left(n + \frac{1}{4}\right)\pi i \qquad (n = 0, \pm 1, \pm 2, \ldots);$$

    (b) the set of values of $\log\left(i^2\right)$ is *not* the same as the set of values of $2\log i$.

(5) Use the definition

$$z^c = \exp\left(c\log z\right) \qquad z \neq 0,$$

to show that $\left(-1 + \sqrt{3}i\right)^{3/2} = \pm 2\sqrt{2}$.