SCHOOL OF MATHEMATICS AND PHYSICS

MATH3401
Problem Worksheet
Semester 1, 2025, Week 5

---

(1) Show that the limit of the function

$$f(z) = \left(\frac{z}{\bar{z}}\right)^2$$

as z tends to 0 does not exist.

Hint: Do this letting nonzero points $z = (x, 0)$ and $z = (x, x)$ approach the origin.

(2) Find $f'(z)$ when

(a) $f(z) = \dfrac{z-1}{2z+1}$, $(z \neq -1/2)$;

(b) $f(z) = \dfrac{(1+z^2)^4}{z^2}$, $(z \neq 0)$.

(3) Determine where $f'(z)$ exists and find its value when

(a) $f(z) = \dfrac{1}{z}$;

(b) $f(z) = x^2 + iy^2$.

(c) $f(z) = z\,\mathrm{Im}(z)$.

(4) Show that each of these functions is differentiable in the indicated domain of definition, and also find $f'(z)$:

(a) $f(z) = \dfrac{1}{z^4}$, $z \neq 0$;

(b) $f(z) = \sqrt{r}e^{i\theta/2}$, $(r > 0, \alpha < \theta < \alpha + 2\pi)$.

(5) Show that each of these functions is nowhere analytic:

(a) $f(z) = xy + iy$;

(b) $f(z) = 2xy + i(x^2 - y^2)$;

(c) $f(z) = e^y e^{ix}$.