SCHOOL OF MATHEMATICS AND PHYSICS

MATH3401
Problem Worksheet
Semester 1, 2025, Week 6

---

(1) Using the appropriate definition of limits involving infinity, show that

(a) $\displaystyle\lim_{z\to\infty} \frac{4z^2}{(z-1)^2} = 4;$

(b) $\displaystyle\lim_{z\to 1} \frac{1}{(z-1)^3} = \infty;$

(c) $\displaystyle\lim_{z\to\infty} \frac{z^2+1}{z-1} = \infty.$

(2) Use the Wirtinger operator

$$\frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$$

to show that if the first-order partial derivatives of the real and imaginary components of a function $f(z) = u(x,y) + iv(x,y)$ satisfy the Cauchy-Riemann equations, then

$$\frac{\partial f}{\partial \overline{z}} = \frac{1}{2}\left[\,(u_x - v_y) + i\,(v_x + u_y)\,\right] = 0$$

Thus derive the *complex form* $\partial f/\partial \overline{z} = 0$ *of the Cauchy-Riemann equations.*

(3) Determine which of the following functions $f(z)$ are entire and which are not? Justify your answer. If $f(z)$ is entire, find $f'(z)$.

(a) $f(z) = \dfrac{1}{1+|z|^2};$

(b) $f(z) = (x^2 - y^2) + 2xyi;$

(c) $f(z) = (x^2 - y^2) - 2xyi.$

Suggestion: Try to use the Wirtinger operator.

(4) Find the derivatives of the following functions in an appropriate domain:

(a) $f(z) = z\,\mathrm{Log}\,z;$

(b) $f(z) = \mathrm{Log}\,(z+1).$