SCHOOL OF MATHEMATICS AND PHYSICS

MATH3401
Problem Worksheet
Semester 1, 2025, Week 7

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(1) Evaluate the following integrals:

(a) $\displaystyle\int_1^2 \left(\frac{1}{t} - i\right)^2 dt$;

(b) $\displaystyle\int_0^{\pi/6} e^{i2t} dt$.

(2) Show that if m and n are integers,

$$\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta = \begin{cases} 0, & \text{when } m \neq n, \\ 2\pi, & \text{when } m = n. \end{cases}$$

(3) Evaluate $\int_C f(z)dz$ for $f(z) = (z+2)/z$ and C is

a) the semicircle $z = 2e^{i\theta}$ $(0 \leq \theta \leq \pi)$;

b) the semicircle $z = 2e^{i\theta}$ $(\pi \leq \theta \leq 2\pi)$;

c) the circle $z = 2e^{i\theta}$ $(0 \leq \theta \leq 2\pi)$.

(4) Find the contour integral $\displaystyle\int_C \overline{z} dz$ for

(a) C is the triangle XYZ oriented counterclockwise, where $X = 0$, $Y = 1 + i$ and $Z = -2$;

(b) C is the circle $|z - i| = 2$ oriented counterclockwise.