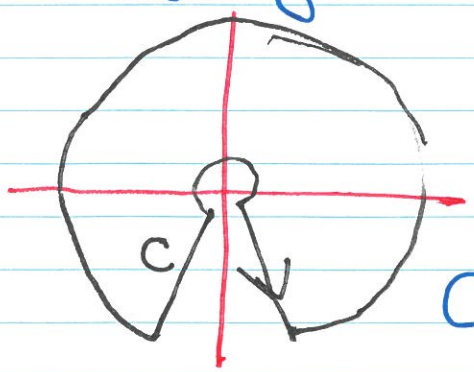
LECTURE 9

RMK: Sometimes, we need to use a different single-valued Log/arg from Log/Arg, e.g., when integrating around a contour such as C, called a keyhole contour.



In this case, we would choose, e.g., to define

$\operatorname{Arg} z$ s.t. $-\frac{\pi}{2} < \operatorname{Arg} z \leq {}^{3\pi}/_{2}$,

particular value of arg.

leading to a new, single-valued Log etc.

Consider an interval o.t.f. (of the form)

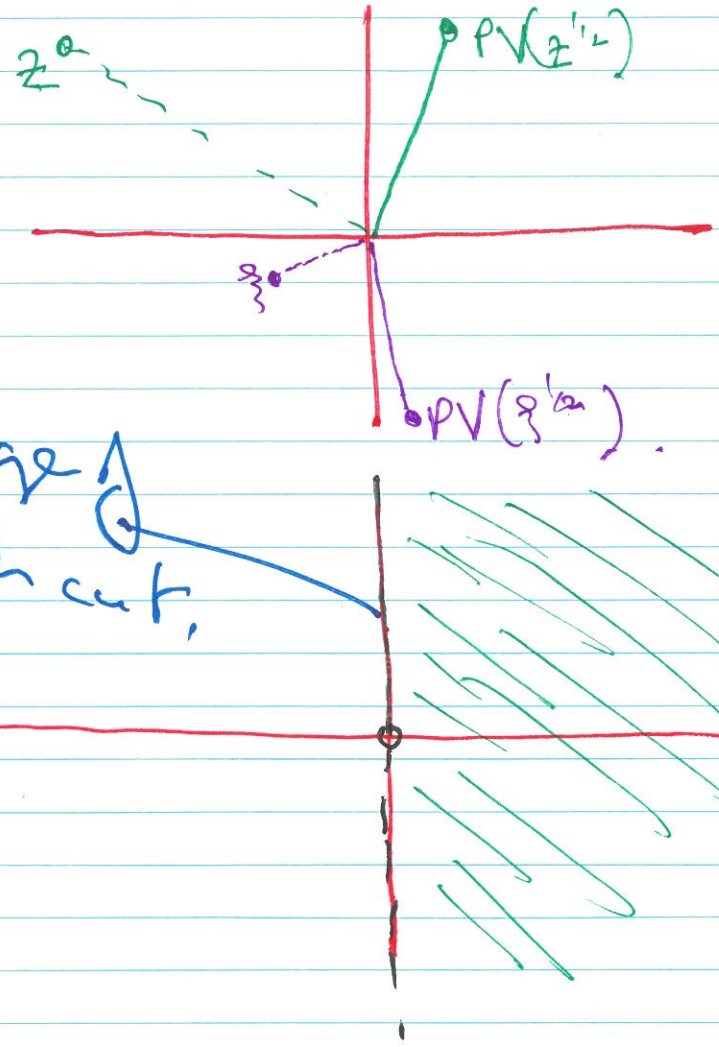
$\alpha \leq \tilde{\theta} < \alpha + 2\pi$ OR $\alpha < \tilde{\theta} \leq \alpha + 2\pi$ for some $\alpha \in \mathbb{R}$. We can define a single valued argument on $\mathbb{C}_*$ with values in that interval, & hence a single-valued logarithm, called a branch of the logarithm. (*)

This then leads to being able to define a single-valued branch of, e.g., $z \mapsto z^{1/2}$.

(*) branch cut $\{z : \arg z = \alpha\} \cup \{0\}$.

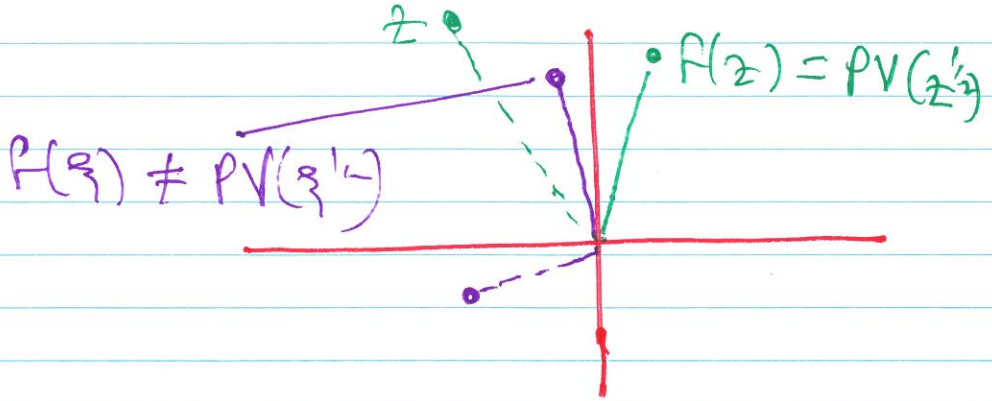
E.g. $PV(z^{1/2})$ $\quad z \mapsto |z|^{1/2} \exp\left(\frac{i \operatorname{Arg} z}{2}\right)$

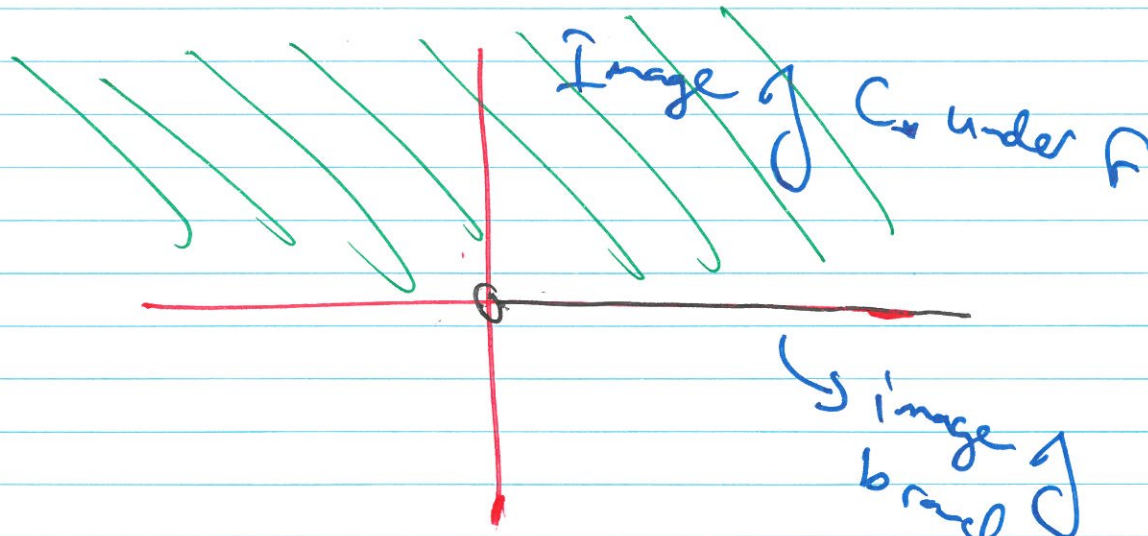
$re^{i\theta} \mapsto \sqrt{r}\, e^{i\theta/2}$ $\quad -\pi < \theta \leq \pi$.



A different branch of $z^{1/2}$

$f: re^{i\theta} \mapsto \sqrt{r}\, e^{i\theta/2}$ $\quad 0 \le \theta < 2\pi$





repeat for $(z-z_0)^{1/2}$, $z_0 \neq 0$. ★