Lecture 18 part 3 : arcs & curves

Jordan arc (a.k.a. simple arc) :
arc s.t. $z(t_1) \neq z(t_2)$ for $t_1 \neq t_2$.
(here $z(t) = x(t) + iy(t)$, $t \in [a,b]$).

Jordan curve (a.k.a. simple closed curve)

1. $z(b) = z(a)$
2. o'wise $z(t_1) \neq z(t_2)$ for $t_1 \neq t_2 \in [a,b]$.

Note exs 2,3,4 from Lec 17 have the same trace.
Exs 2 & 3 are Jordan curves; ex 4 is not.
An arc/curve is called differentiable
if $z'(t)$ exists ($\forall t \in (a,b)$ for an arc,
$\forall t \in [a,b]$ for a closed curve).
If z' is also cts on $[a,b]$, then
$\int_a^b |z'(t)|\,dt$ exists, & defines the
arc length.

<u>Note</u> if $z(t)$ is a parametrisation of the image arc, we can define another one by $t = \Phi(\tau)$, with $\Phi(\alpha) = a$ & $\Phi(\beta) = b$, s.t. $\Phi \in C([\alpha,\beta])$, $\Phi' \in C([\alpha,\beta])$ i.e., $\Phi \in C'([\alpha,\beta])$.

So $z(t) = Z_1(\tau) = z(\Phi(\tau))$

Assume $\Phi'(\tau) > 0 \ \forall \tau$.

$$\int_a^b |z'(t)|\,dt = \int_\alpha^\beta |z'(\Phi(\tau))|\,\Phi'(\tau)\,d\tau$$
$$= \int_\alpha^\beta |Z_1'(\tau)|\,d\tau$$

i.e., arc length is independent of parametrisation.

Next contour, contour integrals