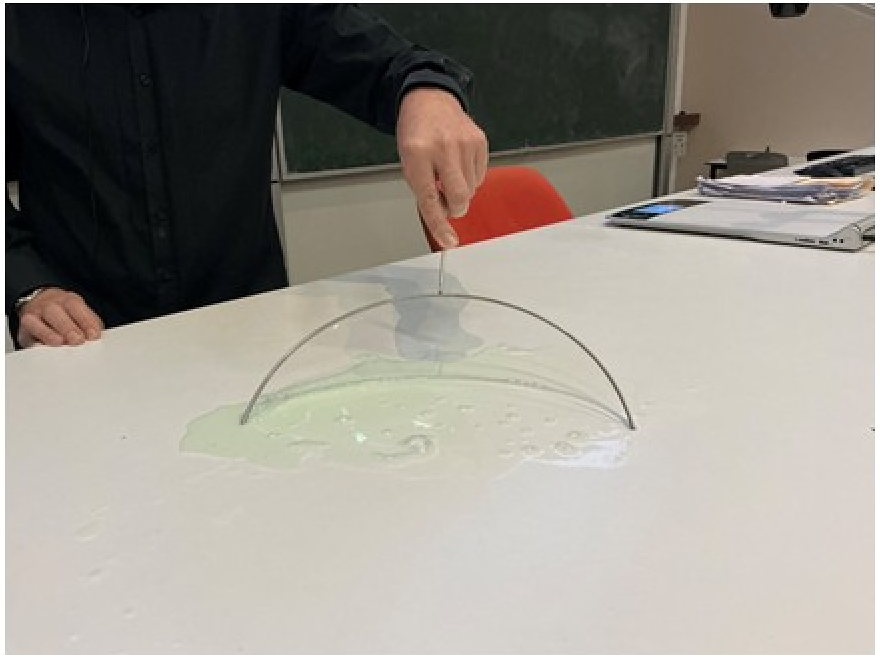
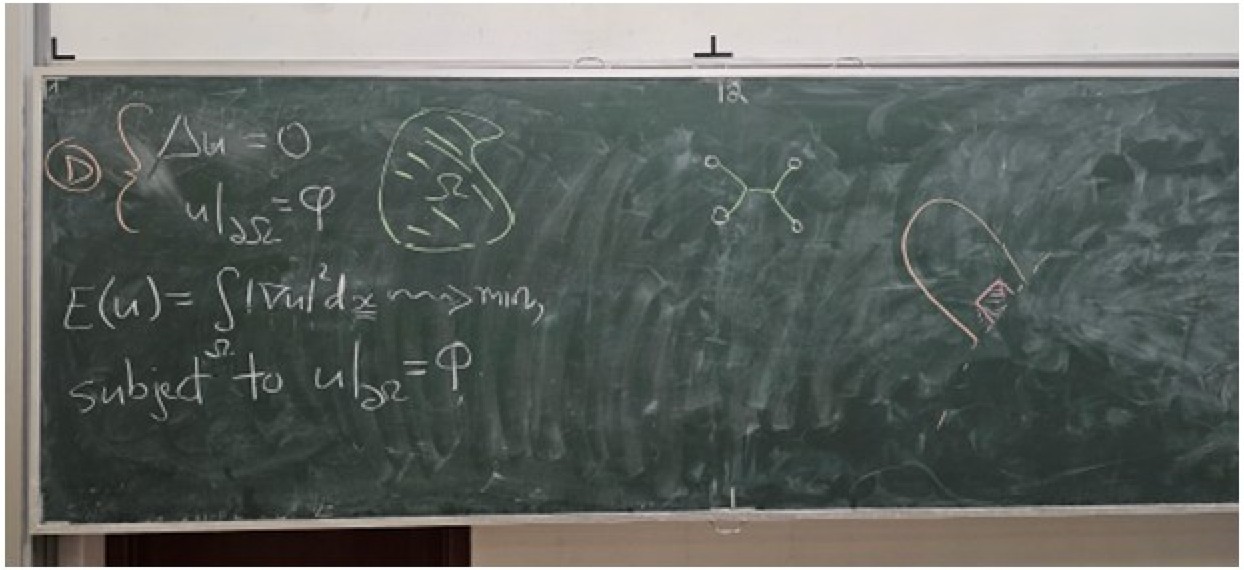
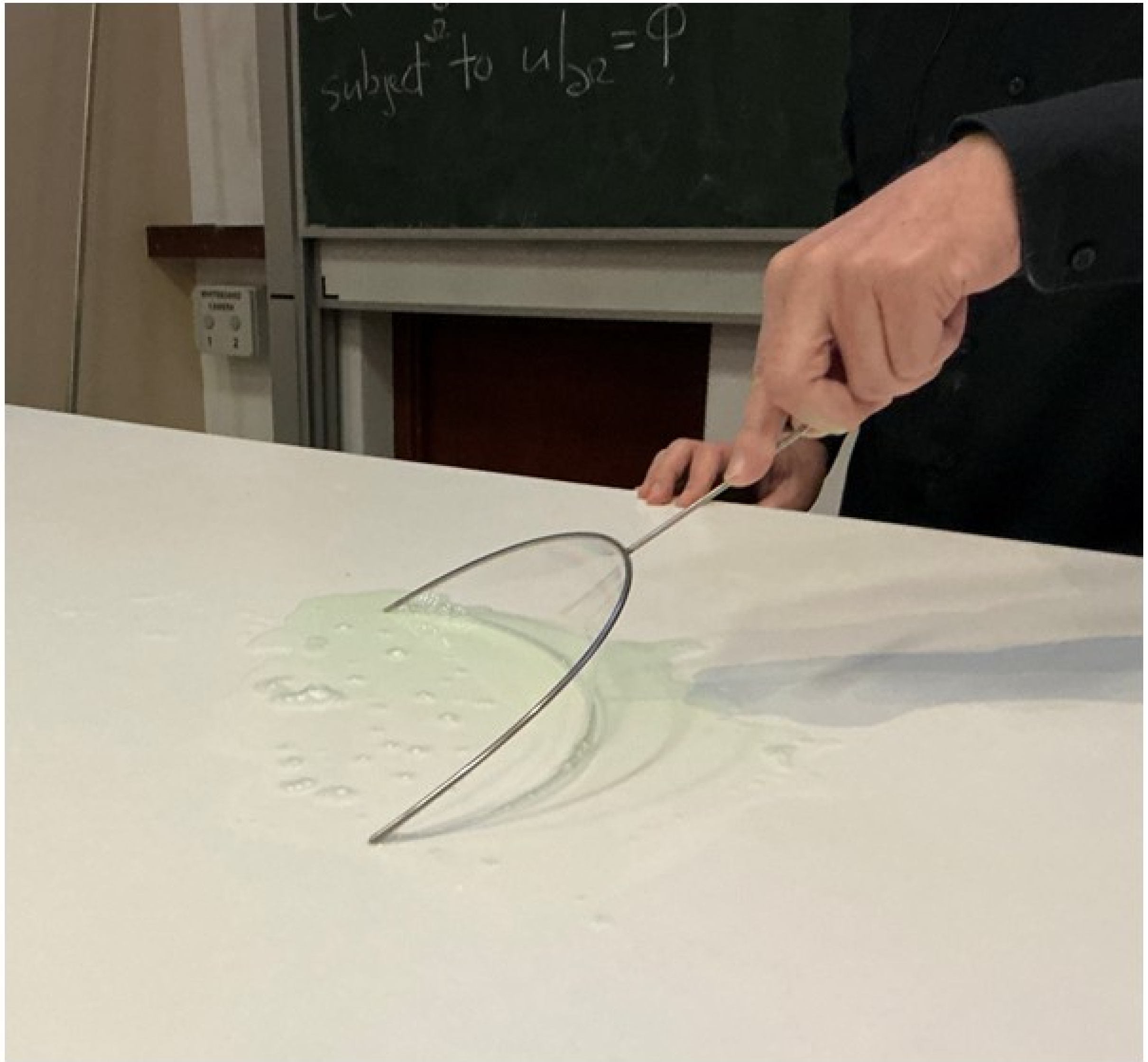




D $\begin{cases} \Delta u = 0 \\ u|_{\partial\Omega} = \varphi \end{cases}$
Ω
12
$E(u) = \int_\Omega |\nabla u|^2 d\underline{x} \rightsquigarrow \min,$
subject to $u|_{\partial\Omega} = \varphi$



subject to $u|_{\partial\Omega} = \varphi$

LECTURE 29 Part 2.

§60 (8Ed §55) Sequences & Series

Formally, a sequence is $f^n: \mathbb{N} \to \mathbb{C}$
(or $\mathbb{N}_0 \to \mathbb{C}$) $n \mapsto z_n$: write $\{z_n\}$.

Limit of a seq.

$$\lim_{n\to\infty} z_n = z \iff \{z_n\} \text{ converges to } z$$

$\iff$ Given $\varepsilon > 0$ $\exists N \in \mathbb{N}$ s.t. $n > N \Rightarrow$
$|z - z_n| < \varepsilon$.

Formally, a series $\sum_{n=0}^{\infty} z_n$, $z_n \in \mathbb{C}$ converges as a series $\iff$ the associated sequence of partial sums $\{S_n\}$ converges as a sequence.

Here $S_n = \sum_{k=0}^{n} z_k$.

Typical q^n: does $\sum z_n$ converge.

Test for NO: nth term test.

$$\sum z_n \text{ converges} \Rightarrow z_n \to 0$$
$$\not\Leftarrow \quad \text{e.g. } \sum\left(\tfrac{1}{n} + 0i\right) \text{ diverges.}$$

RMK: $\{z_n\}$ is bdd say

$$\boxed{\exists M \text{ s.t. } |z_n| < M \ \forall n.}$$

convergent $\Rightarrow$ bdd

$\not\Leftarrow$ e.g. $\{(-1)^n + 0i\}$.

<u>Def^n</u>: the series $\sum z_n$ converges absolutely say $\sum |z_n|$ converges.

Abs. convergence $\Rightarrow$ convergence

$\not\Leftarrow$ e.g. $\left\{\frac{(-1)^n}{n} + 0i\right\}_n$.

Given $\sum_{n=0}^{\infty} z_n$, set $S_n = \sum_{k=0}^{n} z_k$ as above, & write $\rho_n = \sum_{k=n+1}^{\infty} z_k$. ($\rho$ is the tail/remainder).

Then $S_n \to S \Leftrightarrow \rho_n \to 0$. ☆.
some

<u>Applic</u>: Claim $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$ for $|z|<1$. (*)

<u>Pf</u>: analogous to the proof in $\mathbb{R}$.

geometric series.

<u>As in $\mathbb{R}$</u>:

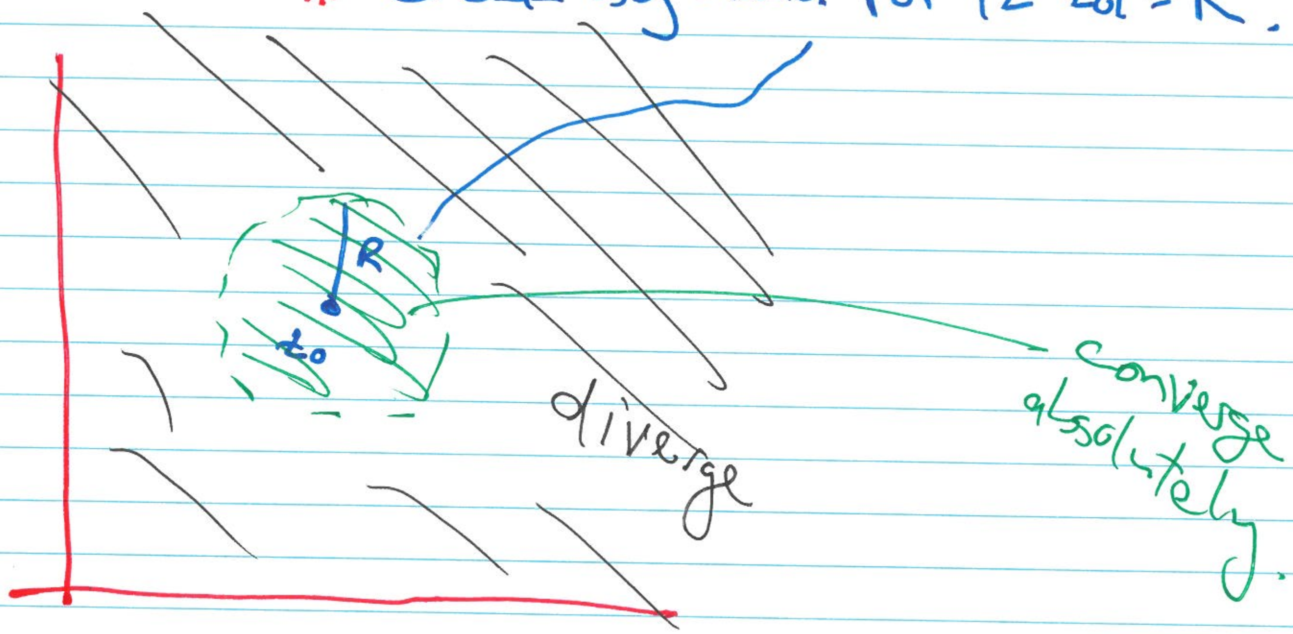
* If $\sum a_n$ & $\sum b_n$ converge, then so do $\sum(a_n \pm b_n)$ to $\sum a_n \pm \sum b_n$.
* For $\lambda \in \mathbb{C}$, $\sum(\lambda a_n)$ converges to $\lambda \sum a_n$.

Power series centred at z_0:

series o.t.f. $\sum_{n=0}^{\infty} a_n (z-z_0)^n$, $a_n \in \mathbb{C}$.

Series will * converge ~~uniformly~~ absolutely for $|z-z_0| < R$, where R is the radius of convergence

* diverge for $|z-z_0| > R$
* check by hand for $|z-z_0| = R$.



* $R = \infty$ converge on $\mathbb{C}$.
* $R = 0$ converge at z_0.