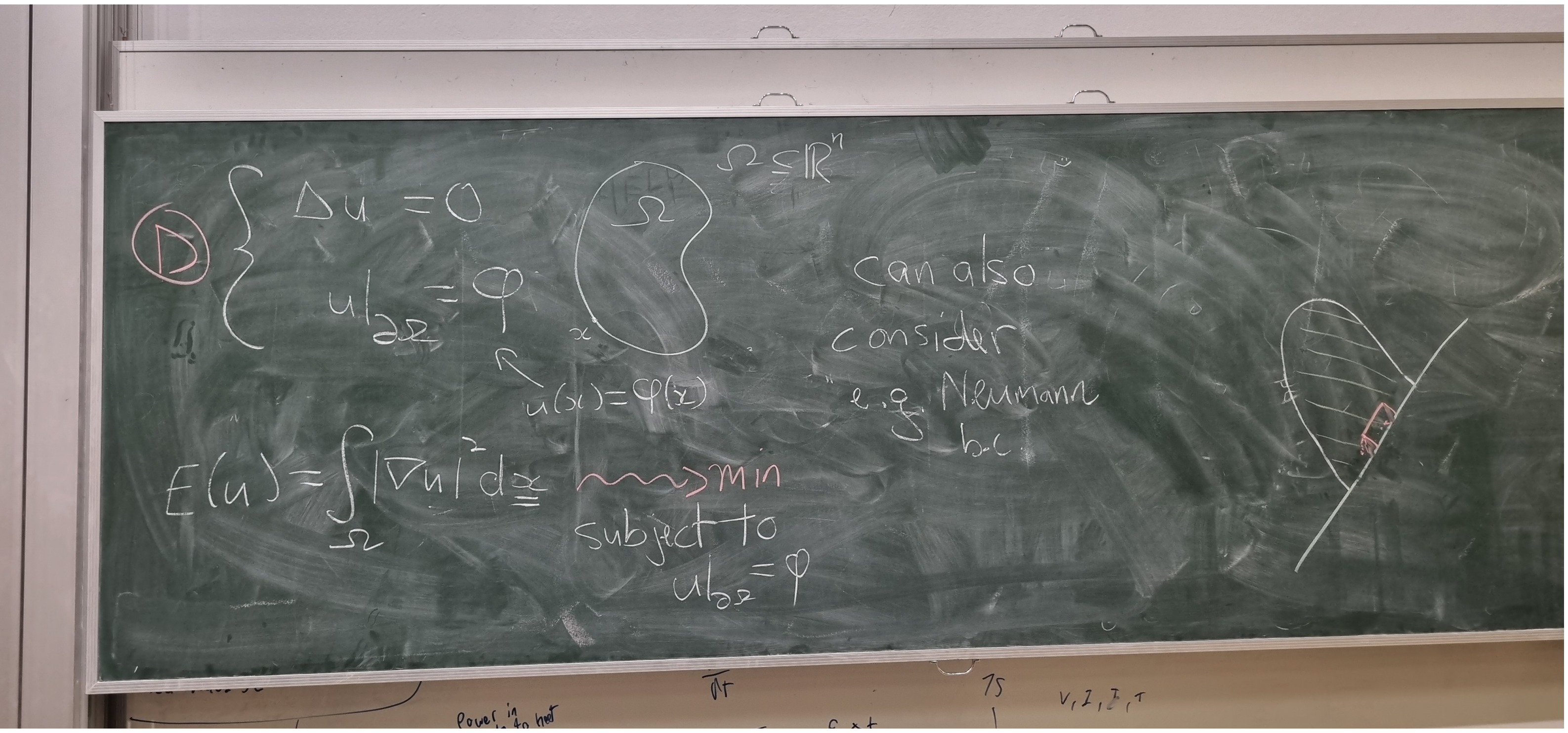
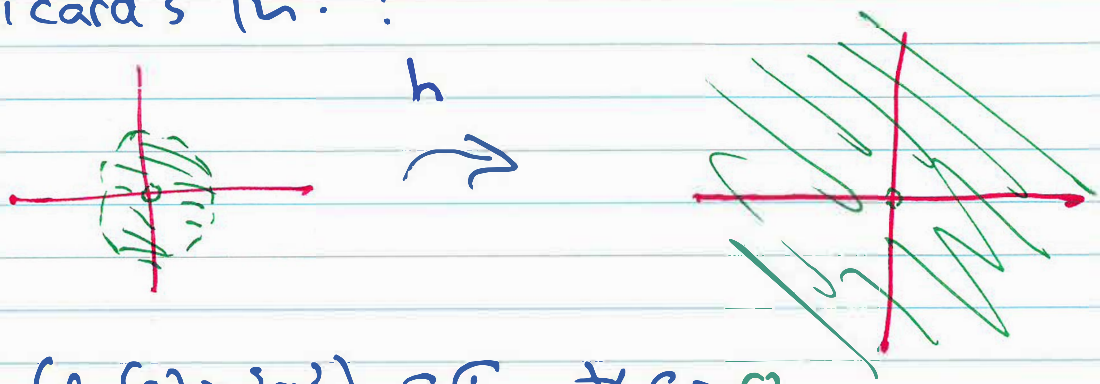

D
$\Delta u = 0$
$u|_{\partial\Omega} = \varphi$
$\Omega \subseteq \mathbb{R}^n$
Ω
x
$u(x) = \varphi(x)$
can also
consider
e.g. Neumann
b.c.
$E(u) = \int_\Omega |\nabla u|^2 \, d\underline{x} \rightsquigarrow \min$
subject to
$u|_{\partial\Omega} = \varphi$

LECTURE 30

$h(z) = e^{1/z}$: analytic on $\mathbb{C}_*$, isolated sing at 0, $\operatorname*{res}_{z=0} h(z) = 1$, noting

$$h(z) = \sum_{n=0}^{\infty} \frac{1}{n!\, z^n} \quad \text{on } \mathbb{C}_*.$$

Picard's thm:



$h(B_\varepsilon(0) \setminus \{0\}) = \mathbb{C}_* \quad \forall\, \varepsilon > 0$

Note no solⁿ to $e^{1/z} = 0$.

Ex 4: $\gamma(z) = \dfrac{1}{1 - 1/z}$ analytic on $\mathbb{C} \setminus \{0,1\}$

compositions of rational fⁿs, isolated sing at 0, 1.

For $z \neq 0, 1$:

$$\gamma(z) = \frac{z}{z-1} = \frac{-z}{1-z}$$

$$= -z(1 + z + z^2 + \cdots) \quad 0<|z|<1$$

$$= -z - z^2 - z^3 - \cdots$$

$\Rightarrow$ removable sing at 0 (no b's in the Laurent series): set $\gamma(0) = 0$ to extend γ to an analytic fⁿ on $\{z : |z| < 1\}$

$\underline{z=1}$ ✗

§82-83 (8Ed §75-76) Zeros & Poles

f analytic at z_0: f has a zero of order m at z_0 if $\begin{cases} f^{(j)}(z_0)=0 & j=0,\dots,m-1 \\ f^{(m)}(z_0)\neq 0. \end{cases}$

E.g. $f(z)=(z-i)^4(z-4)$ has a zero of order 4 at i & a zero of order 1 (a.k.a. simple zero) at $z=4$.

§82 Th^m 1 f analytic at z_0 has a zero of order m at z_0 $\Leftrightarrow$ $f(z)=(z-z_0)^m g(z)$, where g is analytic at z_0, & $g(z_0)\neq 0$.

§83 Th^m 1 Suppose p & q are analytic at z_0, $p(z_0)\neq 0$, q has a zero of order m at z_0. Then p/q has a pole of order m at z_0.

Ex: $p(z)=1$, $q(z)=z(e^z-1)$.

p/q has an isolated sing at 0 (quotient of analytic f^{n}s. p is analytic & non-zero on $\mathbb{C}$.

q is entire & $q(0)=0$;

$q'(0)=0$;

$q''(0)=2\neq 0$.

Th^m $\Rightarrow$ p/q has a pole of order 2 at 0.

THM 2 Let p, q be analytic at z_0.
If $p(z_0) \neq 0$, $q(z_0) = 0$, $q'(z_0) \neq 0$, then
p/q has a simple pole by Th^{m} 1, and

$$\operatorname*{res}_{z=z_0} \frac{p}{q}(z) = \frac{p(z_0)}{q'(z_0)}$$

There are higher order analogues, but
they're messy.