LECTURE 38

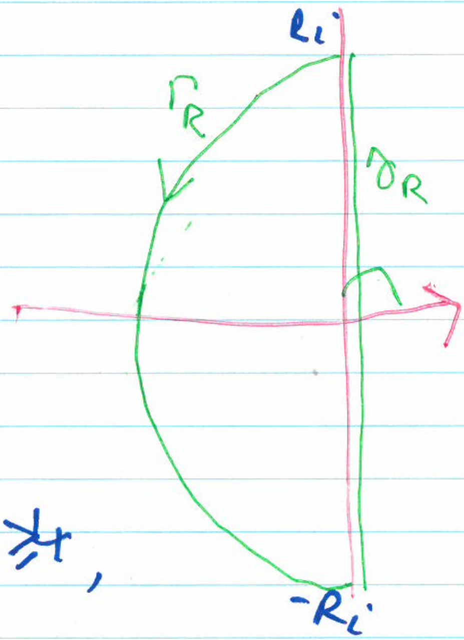
Consultation hours:

Fri. 31/5 1030–1130 &

Fri. 7/6 12–1 pm. or mail.

2020 Q7 Show $h(z) = 2z + 4 + 3e^z$ has precisely one zero in the Left Half Plane.

Sol^n: Consider the contour $C_R = \gamma_R + \Gamma_R$, for R to be specified later.



Put $f(z) = 2z+4$, $g(z) = 3e^z$

Note f, g entire (poly, exp).

On γ_R $z = iy$, so $|f(z)| = |4 + 2iy| \geq 4$,

& $|g(z)| = 3|e^{iy}| = 3 < 4 = |f(z)|$

On Γ_R, $z = x + iy$ with $x < 0$, $|z| = R$.

$\Rightarrow |g(z)| = 3|e^{x+iy}| = 3e^x < 3$ since $x < 0$.

& $|f(z)| = |2z + 4| \geq |2|z| - 4|$ reverse Δ-ineq.

$= |2R - 4|$

$= 2R - 4$ for $R > 2$

> 3 for $R > 7/2$

Hence, for $R > 7/2$, $|f| > |g|$ on Γ_R & hence on C_R. Rouché $\Rightarrow$ f & $f+g = h$ have the same N of zeros inside C_R. f has precisely one zero (at $z = -2$), so h does as well. Letting $R \to \infty \Rightarrow h$ has precisely one zero in LHP.

Final example for Laurent series:
Laurent series for $\frac{1}{\sinh z}$ $= f(z)$ at 0.

f is $1/$entire fn, so is analytic except for the zeros of denom, i.e., $z = n\pi i$, $n \in \mathbb{Z}$.
So 0 is an isolated sing., & f has a Laurent series on $0 < |z| < \pi$

$$f(z) = \frac{1}{\sinh z} = \frac{1}{z + z^3/3! + z^5/5! + \cdots}$$

$$= \frac{1}{z} \cdot \frac{1}{1 + z^2/3! + z^4/5! + \cdots}$$

$$= \frac{1}{z} \cdot \frac{1}{1 - (-z^2/3! - z^4/5! - \cdots)} \quad (*)$$

Using the formula for the sum of a geometric series, for $|z|$ small enough, i.e., s.t. $|z^2/3! + z^4/5! + \cdots| < 1$, there holds:

$$(*) = \frac{1}{z}\left[1 + (-z^2/3! - z^4/5! - \cdots) + (-z^2/3! - z^4/5! - \cdots)^2 + \cdots\right]$$

$$= \frac{1}{z} - z/6 + \frac{7z^3}{360} + \cdots$$

In particular: simple pole at 0, with

$$\operatorname*{res}_{z=0} \frac{1}{\sinh z} = 1 \quad (\text{coeff of } 1/z).$$

Note: can also do by division of power series

Lecture 18 : review covering up to mid-sem.

* line integrals
* contour integrals
* Cauchy-Goursat, Cauchy integral formula
* Formula for derivatives; analytic ⇒ ∞ly diff^ble.
* Morera's th^m.
* Liouville – fund. th^m of algebra
* Conformal mappings.
* Bdy value problems
* Harmonic functions
* harmonic conjugate.
* transformations of bvps.
* Poisson integral formula
* Power/Taylor series/Laurent series
* (analytic ⟺ equal to Taylor series).
* Singularities (isolated/non-isolated)

Classifying isolated sing: * removable
* pole (order)
* essential

* Residue, methods for calculating.

Integration on ℝ using contour integrals & the residue Th^m, properties of PV-integrals,

* M-L th^m, Jordan's Lemma.
* Arg principle ⟶ Rouché's Th^m

not explicitly examinable

* verify conditions of th^ms

watch your algebra, sanity check

remember you are in ℂ.