Assignment Number 1

Problem 1 (3 points) Graph the following regions in the complex plane:
   a) \{z : \text{Re } z \geq (\text{Im } z)^2\};
   b) \{z : \pi/4 < \text{Arg } z \leq \pi\};
   c) \{z : |z - 4i| < \pi\}.

Problem 2 (2 points) Find all complex solutions of the following equations:
   a) \overline{z} = z;   b) \overline{z} = 9/z.

Problem 3 (3 points) Express the following in the form \(x + iy\):
   a) \(\frac{5 + 5i}{3 - 4i + 4 + 3i}\);
   b) all the 3rd roots of 16;
   c) \(\left(\frac{i - 1}{\sqrt{2}}\right)^{1337}\).

Problem 4 (2 points) Suppose that \(z_1\) and \(z_2\) are complex numbers, with \(z_1z_2\) real and non-zero. Show that there exists a real number \(r\) such that \(z_1 = rz_2\).

Problem 5 (4 points) Recall that \(\sqrt{2} \notin \mathbb{Q}\). The set \(\mathbb{Q}\ \text{adjoin } \sqrt{2}\) is defined by \(\mathbb{Q}(\sqrt{2}) = \{p + q\sqrt{2} : p, q \in \mathbb{Q}\}\). Show that \(\mathbb{Q}(\sqrt{2})\) is a subfield of \(\mathbb{R}\).
   Hint 1: remember that \(\mathbb{Q}\) and \(\mathbb{R}\) are fields, and note that \(\mathbb{Q}(\sqrt{2}) \subseteq \mathbb{R}\): hence you will get many, indeed, most of the properties almost free.
   Hint 2: make sure you check closure.

Problem 6 (3 points) A field \(\mathbb{F}\) is called ordered if there exists a distinguished subset \(\mathcal{P} \subset \mathbb{F}\), closed under addition and multiplication, such that the trichotomy law holds, i.e.,
   For every \(x \in \mathbb{F}\), precisely one of the following holds:
   (i) \(x \in \mathcal{P}\);
   (ii) \(x = 0\); or
   (iii) \(-x \in \mathcal{P}\).
   (Note that we can then define an ordering via \(x > y \iff x - y \in \mathcal{P}\)).
   Prove that \(\mathbb{C}\) is not ordered. (Hint: first show that there must hold \(-1 \notin \mathcal{P}\).

Problem 7 (3 points) Write one or at most two paragraphs on the Joukowski transformation. You should also include one relevant picture. Neither your paragraphs nor your picture should be from Wikipedia. Provide references as appropriate.
   (Extra style points if the answer is in verse. Disclaimer: style points are not exchangeable for actual assignment points.)

Due: 11:50AM, Monday, 05/03/2018.

Current assignments will be available at