ASSIGNMENT 3 FOR MATH3402
DUE DATE: 6 MAY 2013

PLEASE SUBMIT IT TO THE ASSIGNMENT BOX,
LEVEL FOUR, PRIESTLEY BUILDING #67

Question 1. *(3 marks)*

(a) Show that the collection of all open intervals $\{(a, b)\}$ *is a basis of* $\mathbb{R}$ *with the standard Euclidean topology.*

(b) Using all basic open sets of $\mathbb{R}$*, prove that* $f(x, y) = x^2 + y^2$ *is continuous function from* $\mathbb{R}^2$ *to* $\mathbb{R}$*.(i.e. Show that* $f^{-1}((a, b))$ *is open in* $\mathbb{R}^2$*.)*

Question 2. *(3 marks)*

Show that the collection of the sets of the form

$$\mathcal{B} = \left\{(x, y) \in \mathbb{R}^2 : a \leq x < b,\ c \leq y < d, \quad \text{with } a, b, c, d \in \mathbb{R},\ a < b, c < d\right\}$$

is a basis for a topology on $\mathbb{R}^2$*.*

(A basis for a topology on X is a collection $\mathcal{B}$ of subsets of X satisfying: (i) for each $x \in X$, there is at least a basic open set B containing x and (ii) if $x \in B_1 \cap B_2$ for two basic open sets B_1, $B_2 \in \mathcal{B}$, then there is a basic open set $B_3 \in \mathcal{B}$ such that $x \in B_3 \subset B_1 \cap B_2$.)

Question 3. *(4 marks)*

(a) By the Heine-Borel Theorem, show that $\mathbb{R}^2$ *is not compact and the sphere*

$$S^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$$

is compact in $\mathbb{R}^3$*.*

(b) Show that $\mathbb{R}^2$ *and* S^2 *is not homeomorphic. (i.e. no continuous bijective function* f *between* $\mathbb{R}^2$ *and* S^2 *such that the inverse function* f^{-1} *is continuous).*