MATH4104: quantum nonlinear dynamics.
Exercise 1, 2009.
Submit on or before Friday September 18.

A simple harmonic is prepared in an initial state for which the position probability amplitude is given by

$$\psi(x, t = 0) = (2\pi\Delta)^{-1/4} e^{-(x-a)^2/4\Delta}$$

Under Schrödinger dynamics, this evolves to

$$\psi(x, t) = e^{-i\omega t/2} \sum_{n=0}^{\infty} c_n u_n(x) e^{-i\omega n t}$$

where the states of definite energy are

$$u_n(x) = (2\pi\Delta)^{-1/4} (2^n n!)^{-1/2} H_n\left(\frac{x}{\sqrt{2\Delta}}\right) e^{-\frac{x^2}{4\Delta}}$$

and

$$c_n = \frac{(a/\sqrt{2\Delta})^n}{\sqrt{n!}} e^{-a^2/4\Delta}$$

1. Prove that the state is periodic.
2. Prove that the average position as a function of time is $\langle x(t) \rangle = a \cos \omega t$