

MATH4104: Quantum Nonlinear Dynamics.

Exercise 2, 2009.

Submit on or before Friday October 16, 5.00pm.

In a hydrogenic atom, the electron is bound in the Coulomb potential of the positive nucleus and has allowed energies given by

$$E_n = -\frac{mc^2\alpha^2 Z^2}{2n^2} \quad n = 1, 2, \dots$$

The negative sign comes from a convention that sets the reference potential to zero for unbound states. Here m is the mass of the electron, c is the speed of light, Z is the atomic number, and the fine structure constant is given by

$$\alpha = \frac{e^2}{\hbar c} \approx 1/137.$$

where e is the electronic charge. The integer n is called the principle quantum number.

A Rydberg wave packet is a non stationary state created by laser excitation of a hydrogen-like atom in which the average energy of the state is determined by an average principle quantum number which is substantially greater than unity, as high as $n = 50$ in some cases. In such a state, for short times, the electron follows an almost classical Keplerian orbit with an average radial distance from the nucleus that is substantially greater than what is found for the ground state. However, as we expect for a nonlinear oscillator, after two or more periods of oscillation, the wave packet departs from the classically expected motion to exhibit sequences of revivals and fractional revivals.

Estimate the time scale for revivals, T_{rev} , for a Rydberg wave packet with average principle quantum number $\bar{n} = 20$ in a hydrogen atom with $Z = 1$.