MATH4104 Assignment 1 (worth 10 %) 2009

In this assignment you will investigate the classical dynamics of a driven nonlinear oscillator given by the following $1\frac{1}{2}$ d.o.f Hamiltonian

$$H(q,\,p,\,t) = H_0(q,\,p) + \epsilon H_1(q,\,p,\,t)$$

where

$$H_0(q,\,p) = \frac{1}{1 + \frac{p^2}{2} + \omega_0^2 \frac{q^2}{2}} \qquad \text{and} \qquad \begin{array}{ll} \text{a)} & H_1 = H_{1a} = q\cos t \\ \text{b)} & H_1 = H_{1b} = q^2 \cos t \end{array}$$

Question 1.

Calculate action angle variables $(\theta,\,I)$ for the unperturbed system, i.e. the system above with $\epsilon = 0$. Use this to obtain a formula for the nonlinear frequency, $\omega(H,\,\omega_0)$ as a function of the energy (H) and ω_0 and/or, $\omega(I,\,\omega_0)$ as a function of the action I and ω_0. Give a plot of $\omega(H,\,\omega_0)$ and/or $\omega(I,\,\omega_0)$ and comment on the role played by ω_0.

Question 2.

Now consider the perturbed systems with ϵ nonzero and small and
a)$H_1(q,\,t) = H_{1a}(q,\,t) = q\cos t$ and
b)$H_1(q,\,t) = H_{1b}(q,\,t) = q^2 \cos t$.

Use perturbation theory to find the position and width of the first and second order resonances in the classical Poincare map in each case. Determine the parameter range for which each resonance exists and investigate their presence numerically. Discuss the extent and level of chaos in each system.

You may use any program you like (matlab, mathematica, dstool etc) to analyze the system numerically. (In these systems the Poincare map is simply a stroboscopic map.)

Question 3.

Compare and contrast the two models H_{1a} and H_{1b}. Discuss the similarities and differences in the resonances in each case and comment on how these reflect the symmetries of the original Hamiltonians.