INTERNAL STUDENTS ONLY

THE UNIVERSITY OF QUEENSLAND

CANDIDATES MUST NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

First Semester Examination, June, 2003

MATH4104

ADVANCED HAMILTONIAN DYNAMICS & CHAOS

(Unit Courses, Inf. Tech.)

Time: **TWO** for working

Ten minutes for perusal before examination begins

Check that this examination paper has 15 printed pages!

CREDIT WILL BE GIVEN ONLY FOR WORK WRITTEN ON THIS EXAMINATION PAPER!
Students should attempt all questions.
The questions each carry 10 marks and part marks are as indicated.
The exam paper is a total of 60 marks.
Calculators allowed.
Check that this examination paper has 15 printed pages!

FAMILY NAME (PRINT): ______________________

GIVEN NAMES (PRINT): ______________________

STUDENT NUMBER: | | | | | | | | |

SIGNATURE: ______________________

EXAMINER'S USE ONLY			
QUESTION	MARK	QUESTION	MARK
1		4	
2		5	
3		6	
TOTAL MARKS			

Q1. (a) Derive Hamilton's equations of motion from Lagrange's equations of motion.

(3 marks)

Question 1 continued on next page.

Q1. (b) Show that the following Hamiltonians are integrable and give their integrals of motion.

(i)
$$H(q,\, p,\, t) = \frac{p^2}{2} + \cos(q) + e^{-t}$$

(ii)
$$H(\theta_1,\, \theta_2,\, I_1,\, I_2) = I_1^2 + 2I_1 I_2 + I_2^2 + I_1 I_2 \cos(\theta_1 - 3\theta_2)$$

(7 marks)

Question 2 on next page. TURN OVER

Q2. Give the primary resonance conditions for the following Hamiltonian.

$$H(\theta_1,\, \theta_2,\, I_1,\, I_2) = 2I_1^2 + I_2^2 + \epsilon(I_1^2 \sin(\theta_1)^2 \cos(\theta_2) + I_2^2)$$

If H is fixed as 1, which tori of the unperturbed system, that is the system with $\epsilon = 0$, will be resonant?

Assuming you are not close to resonance give the near identity generating function that removes all the oscillating terms to order ϵ. Hence give the approximate integrals of motion in the original variables.

(10 marks)

more space for question 2 over page.

Space for question 2

Question 3 see next page.

Q3. (a) Describe what happens as you perturb the following twist map

$$I_{n+1} = I_n$$

$$\theta_{n+1} = \theta_n + 2\pi\omega(I_n), \qquad mod(1)$$

if $\omega = \frac{N}{M}$, given that $\omega(I_n)$ is continuous function and the map remains area preserving. **(5 marks)**

Question 3 continued on next page.

Q3. (b) Show that the following map

$$u_{n+1} = u_n - A\sin(2\pi\phi_n)$$

$$\phi_{n+1} = \phi_n + (u_{n+1})^2 \quad mod(1)$$

is a product of two involutions and find any lines or curves which are fixed under the involutions of the map. Using this or otherwise investigate the possibility of symmetric period-2 points where one iterate lies on the line $\phi_n = 0$.

(5 marks)

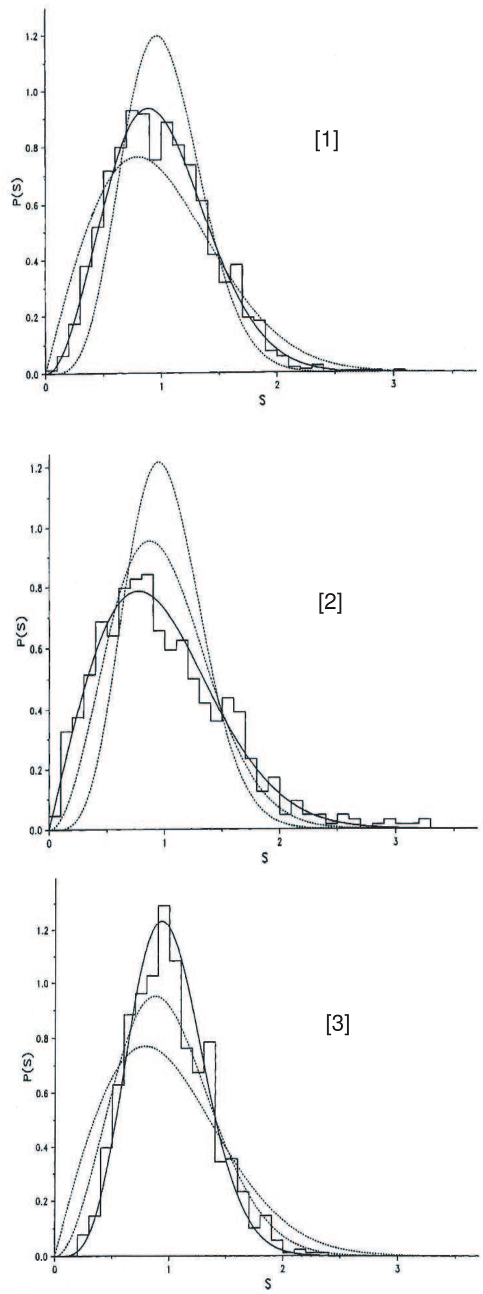
Q4. What is the BGS conjecture ? In less than one page explain what it has to say about the quantum description of systems that are classically chaotic.

(10 marks)

Space for question 4

Question 5 on next page. TURN OVER

Q5. In the figure below, are shown the level spacing distributions for three different anharmonic oscillators.



In each case the solid line is a best fit to one of three random matrix ensemble spacing distributions. Which figure (1,2 or 3) corresponds to the

(a) gaussian orthogonal ensemble. **(3 marks)**

(b) gaussian unitary ensemble. **(3 marks)**

(c) gaussian symplectic ensemble. **(4 marks)**

Write one sentence justifying your choice and discuss the symmetry properties of the Hamiltonian in each case.

space for question 5 on next page.

Space for question 5

Q6 Consider an ensemble of 2×2 real symmetric matrices of the form

$$\begin{pmatrix} \alpha & \beta \\ \beta & \delta \end{pmatrix}$$

where α, β, δ are real and have the joint probability distribution $p(\alpha, \beta, \delta)$ which is invariant under orthogonal transformations of the matrix. That is to say, if

$$\begin{pmatrix} \alpha & \beta \\ \beta & \delta \end{pmatrix} \to \begin{pmatrix} \alpha' & \beta' \\ \beta' & \delta' \end{pmatrix} = O \begin{pmatrix} \alpha & \beta \\ \beta & \delta \end{pmatrix} O^T$$

then $p(\alpha, \beta, \delta) = p(\alpha', \beta', \delta')$.

(a) Prove that, if we require all the elements to be statistically independent, which is to say

$$p(\alpha, \beta, \delta) = p(\alpha)p(\beta)p(\delta) \quad ,$$

then we can take

$$p(\alpha, \beta, \delta) = N \exp[-A(\alpha^2 + \beta^2 + 2\delta^2)] \quad ,$$

where N is a normalisation constant that depends on A. **(3 marks)**

Question 6 continued on next page.

Space for question 6

Question 6 continued on next page.

(b) For this two dimensional matrix ensemble prove that the eigenvalue distribution is given by

$$p(E_1, E_2) = C|E_1 - E_2| \exp[-A(E_1^2 + E_2^2)]$$

where A, C are normalisation constants. **(5 marks)**

Question 6 continued on next page.

(c) For the eigenvalue distribution given in (b) show that the nearest neighbour spacing distribution is given by

$$p(s) = \frac{\pi}{2} s \exp[-\frac{\pi}{4} s^2] \qquad s \geq 0$$

where we have chosen A, C so that $p(s)$ is normalised and has unit mean spacing. **(2 marks)**