MATH7501 Additional Problems for Assignment

Find the limits of the following sequences. If the limit does not exist, explain why.

(a) $\left\{ \cos(2n+1)\frac{\pi}{2} \right\}_{n=1}^{\infty}$

(b) $\left\{ \frac{\pi^n}{4^n} \right\}_{n=1}^{\infty}$

(c) $\left\{ \frac{n^2+3}{n^3+n^2-1} \right\}_{n=1}^{\infty}$

(d) $\left\{ n \sin \frac{\pi}{n} \right\}_{n=1}^{\infty}$

(e) $\left(1-\frac{1}{2}\right), \left(\frac{1}{2}-\frac{1}{3}\right), \left(\frac{1}{3}-\frac{1}{4}\right), \left(\frac{1}{4}-\frac{1}{5}\right), \ldots$

(f) $(\sqrt{2}-\sqrt{3}), (\sqrt{3}-\sqrt{4}), (\sqrt{4}-\sqrt{5}), \ldots$

Determine if the following series converge or diverge:

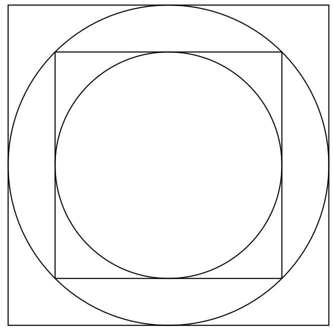
(g) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n\pi)}{n!}$

(h) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n+1} \right)$

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n+\sin(n)}{n^4+n}$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

(k) Consider a square inside which is inscribed a circle, inside which is inscribed a square, inside which is inscribed a circle, and so on, with the outermost square having side length 1. Find the difference between the sum of the areas of the squares and the sum of the areas of the circles.



(l) Let F_n be the n-th Fibonacci number defined by $F_0 = F_1 = 1$, $F_{n+1} = F_n + F_{n-1}$. Determine whether or not the following converges

$$\sum_{n=0}^{\infty} F_n^{-1}.$$

If so, prove carefully and find the limit. If not, state why not.

(m) Let for n a positive integer and $p \in [0, 1]$ let,

$$P(k\,;\,n, p) = \binom{n}{k} p^k (1-p)^{n-k}.$$

Assume now that there is a sequence $\{p_n\}$ such that $n\,p_n = \lambda$ for some $\lambda > 0$ and find an expression for the limit,

$$\lim_{n\to\infty} P(k\,;\,n, p_n).$$

Hint: Look up “Poisson Limit Theorem” or “Poisson approximation to binomial”.