Carry out 6 out of these 8 tasks (please hand in only 6):

1. Use Mathematica to plot the the function $f(x) = \frac{\cos(x)}{|x|}$ over an "interesting" range of values of x. What is the maximal value of $f(x)$? Is the function an even function, an odd function, neither or both?

2. The sample standard deviation of a collection of observations $x_1, \ldots, x_n$ is,

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}},$$

   where $\bar{x}$ is the sample mean. Derive a mechanism for computing the sample standard deviation on-line in a recursive manner that does not require memory storage to grow with n.

3. Consider the on-line document "20 methods of the data scientist and the mathematics behind them". Choose 3 methods from the methods 7–20 and look up what they mean on the web and similar resources. Summarizes the applicability of the methods that you chose, allowing 1-3 paragraphs for each method.

4. Prove De Morgan's law (for n events) by induction.

5. Say you are distributing k balls into n distinct bins where the balls are indistinguishable and there isn't a limit on how many balls fit in a bin. Find an expression for the number of possibilities for this and explain how you got it.

6. Prove:

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

7. Solve question 1 from last year's practice exam:
   https://courses.smp.uq.edu.au/MATH7501/2018/Practice2018.pdf

8. Solve question 1 from last year's exam:
   https://courses.smp.uq.edu.au/MATH7501/2018/2018Final.pdf