1. Consider the matrix:

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix},$$

and the vectors,

$$X = \begin{bmatrix} x \\ y \end{bmatrix}, \qquad \text{and} \qquad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$

(i) What is the matrix $A + A^T$? is it a symmetric matrix?

(ii) What is the matrix A^2 (that is AA)?

(iii) Use Mathematica to evaluate the $(1, 2)$ entry of A^{10}.

(iv) Write out the matrix A^{-1} and state when it exists.

(v) Prove that $AA^{-1} = I$.

(vi) Write out the individual equations in the system of equations for x and y in:

$$AX = b.$$

(vii) Assume that A is non-singular (that is $\det(A) \neq 0$ and the matrix inverse exists). Write the explicit solution to x and y by considering $X = A^{-1}b$.

2. Consider a vector c, a vector d and an $n \times m$ matrix A. Consider the expression $c' A\, d$.

(i) What dimensions do c and d need for the expression to be valid?

(ii) Write out a (double sum) expression for $c' A\, d$ in terms of c_i, $A_{i,j}$ and d_j.

(iii) What if c and d are vectors of 1's. What is the meaning of $c'A$? How about the meaning of $A\, d$? How about $c'Ad$?

3. Prove that $(AB)^T = B^T A^T$ for two matrices A and B with dimensions that allow the multiplication AB.

4. Consider the following Mathematica code:

```
n = 100;
A := Table[RandomReal[], {n}, {n}];
MatrixPlot[A]
ones = Table[{1}, {n}];
s := (Transpose[ones].A.ones)[[1, 1]]
Table[s, {5}]
Histogram[Table[s, {1000}]]
```

Explain in detail what each line does and reason why the output is as it is.

5. Read the start of Section 8.1, dealing with polynomial interpolation:

`https://people.smp.uq.edu.au/YoniNazarathy/julia-stats/StatisticsWithJulia.pdf`

Try to reproduce Figure 8.1 (polynomial interpolation) in Mathematica by using an inverse of the Vandermonde matrix.