1. Assume that you only know: (1) That derivatives are linear. (2) That the derivative of a constant is 0. (3) That the derivative of x is 1. (4) $\frac{d}{dx}x^2 = 2x$. (5) The product rule. (6) The chain rule.
   Use (1)-(6), or a subset to obtain each of the following:
   (i) $\frac{d}{dx}x^4$ (do it using the product rule).
   (ii) $\frac{d}{dx}x^4$ (do it using the chain rule).
   (iii) The quotient rule for derivatives.
   (iv) $\frac{d}{dx}x^{-7}$.
   (v) $\frac{d}{dx}(x+5)^2$ (do it using based on (1)-(4)).
   (vi) $\frac{d}{dx}(x+5)^2$ (do it using the chain rule).

2. Provide a detailed geometric explanation of why $\frac{d}{dx}\sin(x) = \cos(x)$.

3. Consider the standard normal density function, $f(x) = Ke^{-x^2/2}$ where $K = 1/\sqrt{2\pi}$.
   (i) Calculate $f'(x)$.
   (ii) Calculate $f''(x)$.
   (iii) Plot $f(x), f'(x)$ and $f''(x)$ on the same plot with the x-axis range suitably chosen.
   (iv) Argue why $f(x)$ is maximized at $x = 0$ using the first and second derivative.
   (v) Determine the location of the inflection points of $f(x)$ using $f''(x)$. Explain what this means.
   (vi) Take now $g(x) = \frac{1}{\sigma}f\left(\frac{x-\mu}{\sigma}\right)$ with "mean" $\mu \in \mathbb{R}$ and "standard deviation" $\sigma > 0$. Repeat (i), (ii) and (v) for $g(x)$.

4. Assume you have a supply of one kilometer of fencing. You wish to use it to enclose an area with a rectangle, maximizing the area. Formulate this problem as a one variable optimization problem and find the optimal shape. Explain your solution.

5. Consider a continuous function $f(x)$. Assume it has a single root $x^* \in [-1, 1]$, i.e. $f(x^*) = 0$. Write Mathematica code, implementing the bisection method, determining x^* to within accuracy of 10^{-6}. Try your code on $f(x) = \frac{\sin(x-3.5)}{x-3.5}$.

6. Given data points, $x_1, \ldots, x_n$ show that the sample mean, $\overline{x} = \frac{1}{n}\sum_{i=1}^n x_i$ is also the minimizer, η of
$$\sum_{i=1}^n (x - \eta)^2.$$

7. Assume that instead, you now seek η that minimizes
$$\sum_{i=1}^n |x - \eta|.$$
   Is the sample mean still the minimizer? If not, can you find the minimizer? (bonus).

8. Let $x(t)$ be the (continuous) population at time $t \geq 0$. Assume that $x(0) = 5$ and you measure at $t = 2.5$ that $x(2.5) = 10$. Consider these two alternative population growth models:
$$\frac{d}{dt}x(t) = \alpha x(t), \qquad \frac{d}{dt}x(t) = \beta x(t)\Big(1 - \frac{x(t)}{\gamma}\Big).$$
   Use numerical means, or any other means to suggest, α, β, γ for these models, such that both models fit the observations at times $t = 0$ and $t = 2.5$. Then determine $x(5)$ and $x(10)$ for both models and contrast the results.