(1) Let α and β be two real numbers and consider the matrices,

$$A = \begin{bmatrix} 1 & \alpha \\ 1 & 1 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} \beta & -\alpha\beta \\ -\beta & \beta \end{bmatrix}.$$

(i) Set $\alpha = -1$ and determine x and y in the system of equations,

$$A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

(ii) Determine the product $A\,A =$

(iii) Determine the product $A\,B =$

(iv) Given a value of α, with $\alpha \neq 1$, for what value of β does $B = A^{-1}$?

(v) Set $\alpha = \frac{1}{2}$ and $\beta = 2$. Determine $A^9 B^8 =$

(2) Let $\mathbf{1}_n$ be an n dimensional column vector of 1's. Determine for which values of α the following series converges:

$$S = \sum_{n=1}^{\infty} \frac{1}{n^\alpha} \mathbf{1}_n' \big(\mathbf{1}_n \mathbf{1}_n' \big) \mathbf{1}_n.$$

(3) Set

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \qquad \text{and} \qquad V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Define the sequence of vectors $V_1, V_2, \ldots$ via $V_{n+1} = A\,V_n$. That is $V_2 = A\,V_1$, $V_3 = AV_2$, etc. Consider now the sequence of Fibonacci numbers $x_0 = 1$, $x_1 = 1$ and $x_{n+1} = x_n + x_{n-1}$ for $n = 1, 2, \ldots$. Determine a matrix B such that

$$x_n = B\,V_n.$$