(1) Let α be a positive real valued parameter and consider the function

$$f(x) = \alpha \sin(x) + x^5 - 5x^3 + 5x - 1.$$

(i) What is $f'(x)$?

(ii) What is $f''(x)$?

(iii) Consider the subset of the positive integers:

$$\{k \ : \ |f^{(k)}(x)| \leq \alpha, \ \forall x \in \mathbb{R}\}.$$

What exactly are the elements of this set?

(iv) Let x^* be an local extremum point of $f(x)$. Is it true that $-x^*$ is also such a point? Briefly explain your answer.

(2) Consider,

$$A = \begin{bmatrix} e^x & x^2 \\ \sin(x) & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} x \\ \sqrt{x} \end{bmatrix}, \qquad U = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(i) What is $U^T AV$?

(ii) What is $\frac{d}{dx} U^T A\, V$?

(iii) What is $\frac{d}{dx} cos(U^T A\, V)$?

(iv) For what values of x does A^{-1} exist?

(v) Assuming A^{-1} exists what is $\frac{d}{dx} U^T A^{-1}\, U$?

(3) You are measuring the time-delay in seconds between events. This is denoted by t. The function $f(t) = t^2e^{-3t}$ describes the likelihood of values of t for $t \geq 0$. That is, the higher $f(t)$, the higher the likelihood for that specific t.

Determine t for which the likelihood is maximal. Use the first and second derivative to argue why this is the maximal point.