1. Let A be the set of all natural numbers that are divisible by 3. Let B be the set of all natural numbers that are less less 100. Let C be the set of all natural numbers that are less than 200. Let D be the set of all even natural numbers. Let E be the set of negative integers that are greater than -10. Write a program that displays the elements of the set.

$$(A \cap B^c \cap C \cap D) \cup E.$$

2. Let n be a non-negative integer and consider the equation,

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

   Prove that this equation holds in at least two different ways.

3. Use a truth table to check if the following logical statement is a tautology.

$$\Big((A \to B) \wedge (B \to \neg A)\Big) \vee (\neg B).$$

   If it isn't a tautology, make a minimal modification to the logical symbols in the statement to make it a tautology.

4. Use induction to prove:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

5. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ with $f(x, y) = ax^2 + bxy + cy^2 + dx + ey$, where a, b, c, d, and e are real parameter values. Try to determine for which parameter values $f(\cdot)$ is onto. Try to determine for which parameter values $f(\cdot)$ one to one. If you can't determine these sets of parameter values fully, partial credit will be given for finding examples of the parameter values that are/are-not onto and one to one. Feel free to plot $f(\cdot)$ (using a 3D or contour plot) for different parameter values.

6. Consider the relation xRy where $y = f(x)$ for any integer valued x and y, and $f : \mathbb{R} \to \mathbb{R}$. Determine the elements of the relation when $f(x) = \cos(x)$. Determine the elements of the relation when $f(x) = x^2$. Try to determine the elements of the relation when $f(x) = \tan(x)$.

7. Search for a classic proof of the fact that there is an infinite number of prime numbers. Write out the steps of the proof in detail and make sure you understand the proof.

8. Consider the basic question of "how many ways can you throw two dice" and read this blog post (written for the general public):
   https://epsilonstream.com/blog/11-15-21-30-or-36-possible-outcomes/

   Generalize this now to 4 dice instead of 2. What are the number values that replace 11, 15, 21, 30, and 36?