1. Consider a sequence of numbers of the form $x_n = r^n$, where $n = 0, 1, 2, ...$ and $r \in \mathbb{R}$.

   (i) Set $S = \sum_{n=0}^{\infty} \alpha x_n$. For what values of r does the series S converges? Find S and prove your answer.
   (ii) For what values of r does the series S diverge?
   (iii) Assume that $r = \frac{1}{2}$ and $\alpha = e$, where e is the Euler's number. Generate a plot that has two curves. Your first curve will depict partial sums S_k versus k where $k = 0, ..., 20$. Show S_k using black circle symbols. Your second curve will depict the sum S for which the series converges when $k \to \infty$. Show this S value as a red solid curve. Label x-axis of your plot as k. Label y-axis of your plot as S_k. Put inset in your plot showing that the circles corresponds to S_k and the red curve corresponds to S. In up to two sentences, explain the convergence of the curve.

2. Consider series

$$S = \sum_{n=2}^{\infty} \frac{e^n}{3^{n+1}}$$

   Is the series S convergent or divergent? If you think the series diverges, explain your reason. If you think the series is convergent, then find the sum S.

3. Use the methods for determining multivariate limits and prove that the following limits do not exist:

   (i)

$$\lim_{(x,y)\to(0,0)} \frac{xy^3 \cos x}{2x^2 + y^6}$$

   (ii)

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz + xz}{x^2 + y^2 + z^2}$$

4. Consider the following function

$$f(x) = \frac{1}{x^2 - x}$$

   on interval (0, 1).

   (i) Find the critical points of $f(x)$.
   (ii) What are the global maximum and minimum of $f(x)$, if exist?
   (iii) Where do the global maximum and minimum of $f(x)$ occur, if exist?
   (iv) Plot $f(x)$ versus x. Show the curve as a black solid curve. Label x-axis of your plot as x. Label y-axis of your plot as $f(x)$. Do your findings of global maximum, global minimum, and critical points agree with your plot? Explain it in up to three sentences.