

1. Let

$$g(x) = (2x - 3)(x^2 - 5)^3$$

$$f(x, y, z) = \ln(2x - y) + e^{2xz}$$

$$h(x) = 2x^3 \sin(x^4)$$

(i) Find dg/dx .

(ii) Find df .

(iii) Find $\int h(x)dx$.

2. Assume that the pressure P of a certain gas is modelled as:

$$P = 8.314 \frac{T}{v}$$

where T is the temperature and v is the volume.

(i) Formulate the change in the pressure over time due to any change in T and v .

(ii) Assume that volume increases from 20 L to 20.5 L and temperature decreases from 300 K to 295 K. What is the change in the gas pressure?

3. Consider a two-dimensional surface $f(x, y) = \sqrt{x^2 + y^2 + 1}$.

(i) Find the direction of the maximum slope of the surface at any point.

(ii) Find the direction of the maximum slope of the surface at point (2, 2).

(iii) Find the slope of the surface at point (2, 2) in the direction of $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$.

4. Consider the Gaussian integral

$$\int_{-\infty}^{\infty} e^{-x^2} dx$$

(i) Evaluate the Gaussian integral. Explain all the steps you take.

(ii) Assume that the range of the integral above becomes 0 to ∞ . Then, evaluate the following integral and justify all the steps you take:

$$\int_0^{\infty} e^{-x^2} dx$$