

Math7501 Quiz 2, Semester 1, 2020

1. Consider the universal set, $U = \{x \in \mathbb{Z}^+ : x \leq 10\}$ and the sets $A = \{x \in U : x > 7\}$ and $B = \{1, 2, 3\}$

(a) Write out all the elements of U explicitly

(b) Write out all of the elements of A explicitly

(c) What is $A \cap B$?

(d) What is $A \cup B$?

(e) What is $A^c \cap B$?

(f) What is $A \times B$?

(g) Write the elements of $\mathcal{P}(B)$?

(h) What is $|\mathcal{P}(B)|$?

2. Consider the following logical expression $(A \vee B) \wedge \neg(A \wedge B)$

(a) Write the truth table for the above expression

(b) Write an expression using only ANDs, ORs and NOTs that is logically equivalent to the above expression

3. Prove $\sum_{i=1}^n (2i - 1) = n^2$

4. Consider the exponential function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = e^x$

(a) what is the domain of f ?

(b) what is the codomain of f ?

(c) what is the range of f ?

(d) is f one to one (injective)?

(e) is f onto (surjective)?

(f) is f invertible? If it is not invertible what is a simple redefinition of f that will make it invertible?