(1) Evaluate each of the following expressions. Fully justify your use of any theorems from class.

(i) $\lim_{x\to\infty} e^{-\frac{1}{x^2}}$.

(ii) $\lim_{x\to 2} \frac{x-2}{\cos(\pi x)+1-x}$.

(iii) $\lim_{x\to 0} f(x)$, where $f(x)$ is any real function such that $x < f(x) < -x$ for all $x < 0$, and $-x^{10} < f(x) < x^5$ for all $x > 0$.

(iv) $\sum_{n=1}^{\infty} \frac{e}{\pi^n}$.

(v) $\lim_{x\to\infty} \frac{x-2}{\cos(\pi x)+1-x}$.
Hint: It is useful to note that for $x > 10$, $-x \leq \cos(\pi x) + 1 - x \leq 2 - x$.

(2) Consider the function

$$f(x) = \begin{cases} 0 & \text{if } x = 0, \\ e^{-\frac{1}{x^2}} & \text{if } x \neq 0. \end{cases}$$

(i) Show that f is continuous at any non-zero value of x.

(ii) Calculate $f'(x)$ for each $x \neq 0$.

(iii) Show that f is continuous at the point $x = 0$.

(3) Consider the real-valued function

$$f(x) = x^3 + 4x^2 - 20x + 1.$$

(i) Calculate $f'(x), f''(x)$.

(ii) Find all of the critical points of f; determine if they are local maxima/minima.

(iii) How many distinct real valued roots does f have? Prove your answer by making reference to the IVT and the MVT.