Markovian (and Deterministic) Dynamic System[1]

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1 Introduction

Let us take the traffic light system as an example. Considered such a traffic light in practice, there are red, green and yellow. Three different colours run in a designed subsequence, red firstly, then green and yellow lastly. Then, this sequence can be seen as a state machine, each time of a colour appearing only depends on the last colour. Described in mathematics notation, x_1 = red, x_2 = green, x_3 = yellow, x_4 = red again, the state x_i is a deterministic pattern, with each state only determined by the last state.

Thus, we can deduce a linear dynamical system that is a simple model for a sequence $x_1, x_2, \ldots$ in which each x_{t+1} is a linear function of x_t:

$$x_{t+1} = A_t x_t, \quad t = 1,2,\ldots$$

The $n \times n$ matrices A_t are called the dynamics matrices. The equation above is called the dynamics or update equation.

This linear dynamical system is sometimes called a Markov model (after the mathematician Andrey Markov). Markov studied systems assumed that future states depend only on the current state, i.e. x_{t+1} depends only on x_t, and not additionally on $x_{t-1}, x_{t-2}, \ldots$

2 Method

2.1 Deterministic Patterns

The deterministic pattern is a system in which no randomness is involved in the development of future states of the system. A deterministic model will thus always produce the same output from a given starting condition or initial state.

2.2 Non-deterministic Patterns

Differentiated from deterministic patterns, non-deterministic systems will give different solution each time for the same input.

2.3 χ^2 test

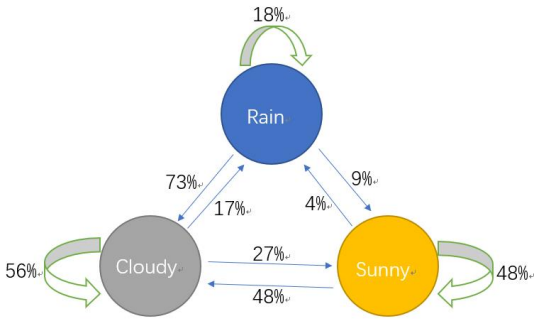
It is used to test independency of events and ensure accuracy of our model.

3 Tool

Editor: Jupyter Notebook; Programming: Python 3; Visualisation: Setosa.io[2].

4. Application

Take 2017 Shanghai's winter weather records as the example. We modelled it with state 1, 2 and 3, representing sunny days, cloudy days and rainy days. The Final results are shown as below, a transition probability matrix visualisation.



[1] The link of video: https://youtu.be/9WpuBwX-Q8Q

[2] http://setosa.io/#/