

Please make sure to follow the hand-in instructions. Also, please present your answers in order, showing the working for each answer. Answering yes/no is not enough. You should rather present an argument or derivation of your answer.

Consider the vectors  $v_1 = [-1 \ 0 \ -1]^T$ ,  $v_2 = [0 \ -1 \ 1]^T$ , and  $v_3 = [0 \ 1 \ 1]^T$ . Let the  $3 \times 3$  matrix  $A$  have columns  $v_1$ ,  $v_2$ , and  $v_3$ . That is  $A = [v_1 \ v_2 \ v_3]$ .

1. Do the vectors  $v_1$ ,  $v_2$ , and  $v_3$  constitute a set of orthogonal vectors?
2. Are the vectors  $v_2$  and  $v_3$  orthogonal?
3. Execute the Gram-Schmidt procedure on the vectors  $v_1$ ,  $v_2$ , and  $v_3$  in this order. Present the output as columns of an orthogonal matrix  $Q$ .
4. Determine  $Q^{-1}$ .
5. What is the nullspace of the matrix  $A$ ?
6. What is the rank of the matrix  $A$ ?
7. Are the vectors  $v_1$ ,  $v_2$ , and  $v_3$  a basis for  $\mathbb{R}^3$ ?
8. Now execute the Gram-Schmidt procedure on the vectors  $v_3$ ,  $v_2$ , and  $v_1$  in the order  $(3 \rightarrow 2 \rightarrow 1)$ . Present the output as the columns of an orthogonal matrix  $M$ .
9. Define the matrix  $B = QM$ . Is  $B$  an orthogonal matrix?
10. Consider now the  $9 \times 9$  block matrix,

$$G = \begin{bmatrix} Q & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & Q \end{bmatrix},$$

where each 0 is a  $3 \times 3$  matrix of zero values. What is the inverse matrix of  $G$ ?