Please make sure to follow the hand-in instructions. Also, please present your answers in order, showing the working for each answer. Answering yes/no is not enough. You should rather present an argument or derivation of your answer. **Tip:** Do NOT waste time on excessive computations because the quiz can be solved without requiring big computations.

1. Consider the vector $u = [1 \;\; 2]^T$ and the vector $v = [1 \;\; 10]^T$. Let the matrix A be the outer product $A = u\,v^T$. Determine the eigenvalues of A.

2. Now set $u = [1 \;\; 2 \;\; 3 \;\; 4 \;\; 5]^T$ and $v = [1 \;\; 10 \;\; 10^2 \;\; 10^3 \;\; 10^4]^T$. With these values determine the eigenvalues of $A = u\,v^T$. Hint: Remember that the sum of the eigenvalues is the trace.

3. Let B be a 3×2 matrix with $\text{rank}(B) = 2$, and set the vector $c = [0 \;\; 1 \;\; 1]^T$. Assume that,

$$B(B^TB)^{-1}B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}.$$

   Let x^* be the value of x that minimizes $||Bx - c||$. Determine the value of Bx^* and the value of $||Bx^* - c||$.

4. Continuing with the same B, prove that B^TB is a positive definite matrix.

5. What are the 3 eigenvalues of $B(B^TB)^{-1}B^T$? Why?

6. Consider the sequence $x(0), x(1), x(2), \ldots$ of vectors in $\mathbb{R}^3$ with

$$x(k+1) = B(B^TB)^{-1}B^Tx(k) - \frac{1}{2}x(k),$$

   for $k = 1, 2, \ldots$. Argue about the value of the limit,

$$\lim_{k\to\infty} ||x(k)||.$$

   Does it depend on $x(0)$? Or is it the same limit for all $x(0)$? What is the limit?