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Student Number

Family Name \_\_\_\_\_

First Name \_\_\_\_\_

**School of Mathematics & Physics**

**EXAMINATION EXAMPLE 1**

Semester One Final Examinations, 2016

**STAT2201 Analysis of Engineering and Scientific Data**

*This paper is for St Lucia Campus students.*

Examination Duration: 120 minutes

Reading Time: 10 minutes

**Exam Conditions:**

This is a Central Examination

This is a Closed Book Examination - no materials permitted

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

**Materials Permitted In The Exam Venue:**

**(No electronic aids are permitted e.g. laptops, phones)**

Calculators - Any calculator permitted - unrestricted

**Materials To Be Supplied To Students:**

none

**Instructions To Students:**

**Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.**

**For Examiner Use Only**

Question Mark

1a	
1b	
1c	
1d	
2a	
2b	
2c	
2d	
3a	
3b	
3c	
3d	
4a	
4b	
4c	
4d	

Total \_\_\_\_\_

## Instructions

The exams consists of 4 questions, 1-4. Each question has four items, a-d.

Within each question:

Item (a) carries a weight of 8 marks.

Item (b) carries a weight of 7 marks.

Item (c) carries a weight of 6 marks.

Item (d) carries a weight of 4 marks.

The total marks in the exam are 100.

Answer ALL questions in the spaces provided. If more space is required, use the back of the PREVIOUS page.

Show all your working and include sketches where appropriate.

Work written in the Formulae and Tables section will NOT be marked.

**Question 1**

You are presented with a sample composed of  $n = 9$  observations  $y = \{y_1, y_2, \dots, y_9\}$  where  $y_i = (i - 5)^2$ .

(a) Calculate the sample mean for this sample.

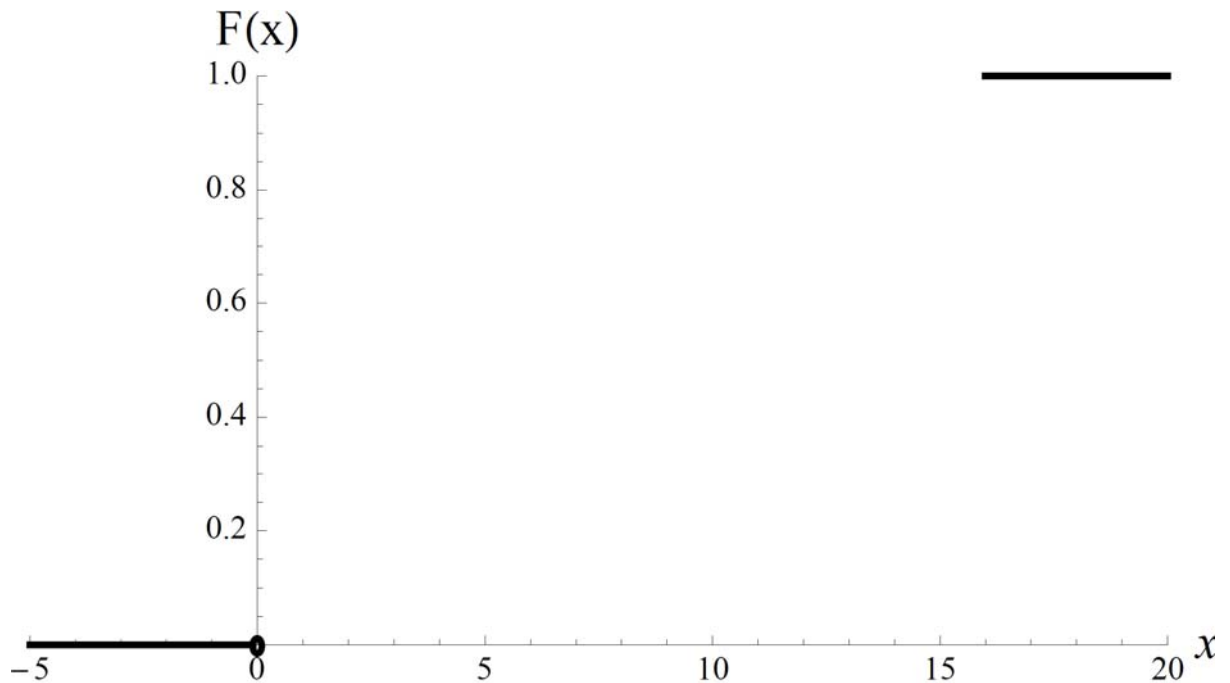
$$\bar{y} = \underline{\hspace{2cm}}$$

(b) Calculate the sample standard deviation.

$$s = \underline{\hspace{2cm}}$$

(c) Sketch the ECDF,  $F(x)$ , for this sample, where

$$F(x) = \frac{1}{n} \sum_{i=1}^n 1\{y_i \leq x\}.$$



(d) Assume now that you are presented with 91 additional observations,  $y_{10}, y_{11}, \dots, y_{100}$ , all equalling exactly 0 (that is  $y_i = 0$  for  $i \in \{10, \dots, 100\}$ ). Calculate the revised sample standard deviation.

$s =$  \_\_\_\_\_

## Question 2

A manufactured device is made out of 5 sub-components, denoted by  $i = 1, 2, \dots, 5$ . The  $i$ th component has a mean weight of  $10i$  grams. The components are assumed to be Normally distributed with (population) variances all equalling 4.

- (a) What is the probability that the weight of a randomly selected  $i$ 'th component exceeds  $10i + 2$ ?

Probability = \_\_\_\_\_

- (b) What is the probability that the total weight of the device exceeds 160 grams?

Probability = \_\_\_\_\_

- (c) A sample of 9 devices is collected. What is the probability that the sample mean is less than 140g?

Probability = \_\_\_\_\_

- (d) Assume instead that the device can be manufactured with an arbitrary number of components,  $n$ . Using the formula

$$\sum_{i=1}^n i = \frac{n(n+1)}{2},$$

find the minimum number of components such that the mean device weight exceeds 300g. Using this, what is the probability that a randomly chosen device weighs less than 290g?

Probability = \_\_\_\_\_

### Question 3

You have obtained a random sample of 15 measurements of the temperature from a computer CPU, operating at full load with a sample mean of 70 degrees and a standard deviation of 5 degrees. A similar independent sample of 15 measurements is obtained at light load with a sample mean of 50 degrees and a sample standard deviation of 4 degrees.

- (a) Calculate and interpret a 95% confidence interval for the mean full load temperature, denoted by  $\mu_f$ , assuming that the measurements are Normally distributed.

Confidence Interval=\_\_\_\_\_

Interpretation: \_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

- (b) Calculate a 95% confidence interval for the difference in mean temperatures,  $\mu_f - \mu_l$ , assuming Normality and equal variances.

Confidence Interval=\_\_\_\_\_

- (c) You wish to test whether the mean temperature at full load is greater than that at the light load, i.e. :  $H_0 : \mu_f = \mu_l$  vs.  $H_A : \mu_f > \mu_l$ . Carry out an appropriate test, assuming  $\alpha = 0.05$ , and state your conclusion.

p-value= \_\_\_\_\_

conclusion: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

- (d) Assume instead that you are producing a new kind of CPU designed to operate at a temperature that is not influenced by a high load. You have carried out the same test as in (c) with separate data, and have failed to reject  $H_0$ , concluding that your new CPU has met its stated goal. Can this be interpreted as there is  $100\alpha\%$  chance that the CPU has not met this goal? Explain why/why not.

*[Your answer should not exceed 4 sentences.]*



### Question 4

You are comparing the strength of 4 types of similar materials and wish to see if the material type affects the strength. You carry out an experiment measuring 10 measurements per material type (a total of 40 strength measurements).

Denoting the mean strengths of material type  $i$  by  $\mu_i$ , you are interested in testing against  $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$ .

After collecting the data, you carry out an ANOVA.

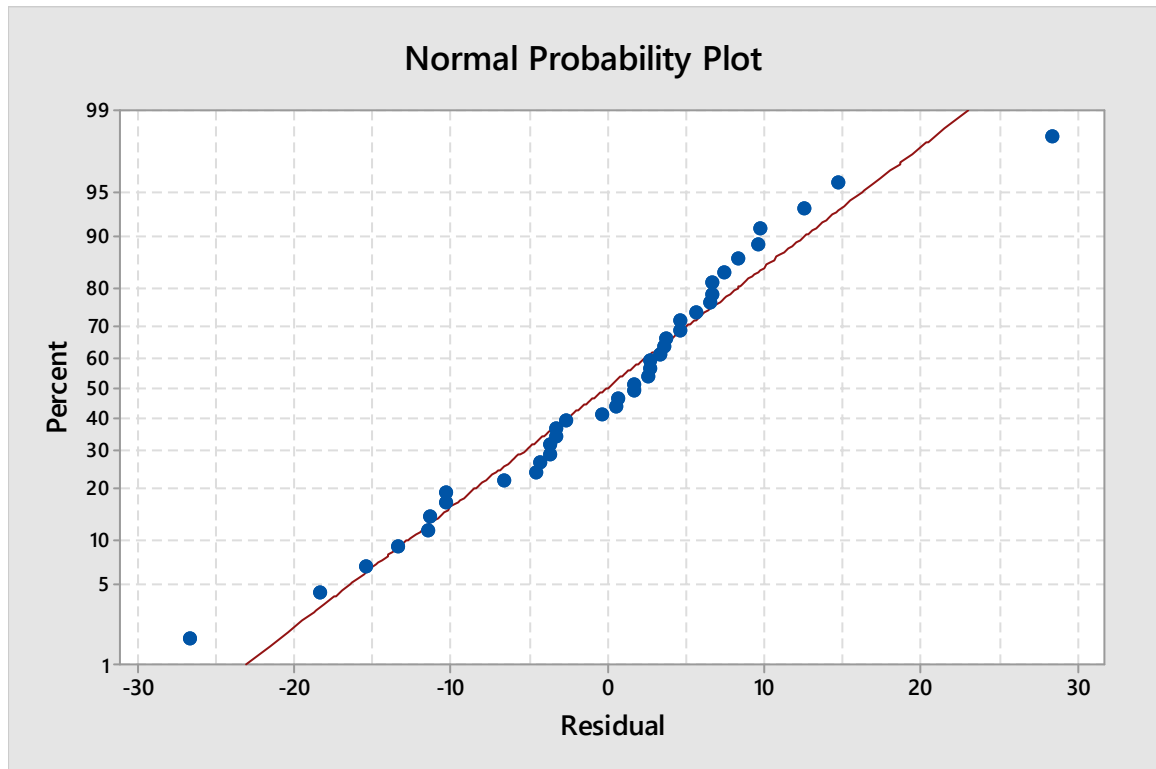
(a) Fill in the blanks in the ANOVA table below :

Source	DF	SS	MS	F-Value	P-Value
Factor			66.03	0.62	0.607
Residual					
Total		4033.1			

(b) Based on the ANOVA, what do you conclude with  $\alpha = 0.1$ ?

- (c) In carrying out the above ANOVA, there are some assumptions made about the distribution of the residuals and their variances.

As an aid to see if these assumptions hold, you are presented with the following plot:



Based on the plot, comment on the validity of the corresponding model assumption:

- (d) It is known that the meaning of MSE is the pooled sample variance over all four types. Assume now that all the means over the four types remain the same, but the sample standard deviations are each scaled down by a factor 10. That is, if the previous sample standard deviation of type  $i$  was  $s_i$ , then it is now  $s_i/10$ . What is the new value of the F-statistic? In which direction does the p-value change?

## Formulae and Tables

### Summary Statistics:

Suppose  $Y_1, Y_2, \dots, Y_n \sim N(\mu_Y, \sigma_Y^2)$  and  $X_1, X_2, \dots, X_m \sim N(\mu_X, \sigma_X^2)$  are two independent samples. The sample means and sample variances are respectively,

$$\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i \quad S_Y^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})^2,$$

$$\bar{X} = \frac{1}{m} \sum_{i=1}^m X_i \quad S_X^2 = \frac{1}{m-1} \sum_{i=1}^m (X_i - \bar{X})^2.$$

Note that,  $\sum_{i=1}^n (Y_i - \bar{Y})^2 = \left( \sum_{i=1}^n Y_i^2 \right) - n\bar{Y}^2$ .

The (two-sample) pooled sample variance is  $S_p^2 = \frac{(n-1)S_Y^2 + (m-1)S_X^2}{n+m-2}$ .

The ECDF function is:  $F(t) = \frac{1}{n} \sum_{i=1}^n 1\{Y_i \leq t\}$ .

### Sampling Distributions:

$$\frac{\bar{Y} - \mu_Y}{\sigma_Y / \sqrt{n}} \sim Z \quad \frac{\bar{Y} - \mu_Y}{S_Y / \sqrt{n}} \sim t_{n-1} \quad \frac{(\bar{Y} - \bar{X}) - (\mu_Y - \mu_X)}{S_p \sqrt{\frac{1}{n} + \frac{1}{m}}} \sim t_{n+m-2} \text{ if } \sigma_X = \sigma_Y.$$

### Confidence Intervals for the Mean:

If  $\sigma_Y$  is known, use  $(\bar{y} \pm z^* \sigma_Y / \sqrt{n})$ .

If  $\sigma_Y$  estimated by  $s_Y$ , use  $(\bar{y} \pm t^* s_Y / \sqrt{n})$ .

### Confidence Intervals for the Difference in Means for Independent Samples:

If  $\sigma_Y$  and  $\sigma_X$  are known, use  $(\bar{y} - \bar{x}) \pm z^* \sqrt{\frac{\sigma_Y^2}{n} + \frac{\sigma_X^2}{m}}$ .

If  $\sigma_Y$  and  $\sigma_X$  are unknown but assumed equal, use  $(\bar{y} - \bar{x}) \pm t^* s_p \sqrt{\frac{1}{n} + \frac{1}{m}}$ .

**Hypothesis Test Basics:**

Type I error: Rejection of the null hypothesis falsely (rejecting the null hypothesis when it is actually true).

Type II error: Non-rejection (retention) of the null hypothesis falsely (not-rejecting the null hypothesis when the alternative hypothesis is true).

$$\Pr(\text{type I error}) = \Pr(\text{reject } H_0 \mid H_0 \text{ holds})$$

$$\Pr(\text{type II error}) = \Pr(\text{not reject } H_0 \mid H_A \text{ holds})$$

Significance level :  $\alpha = \Pr(\text{type I error})$

Power =  $1 - \Pr(\text{type II error})$

**General Hypothesis Test Procedure:**

- i. Write down the null and alternative hypotheses.
- ii. Select an appropriate test statistic for the test and compute it based on data.
- iii. Sketch the distribution of the test statistic and mark the observed value on the plot (also the "opposite" value if the test is two-sided).
- iv. Compute the tail area (or bounds for the tail area), which gives the p-value (multiply the area by 2 for a two-sided alternative hypothesis).
- v. State the conclusion: If computationally possible, report the p-value; otherwise, compare the test statistic with the critical value.

**Hypothesis Tests for the Mean:**

To test against  $H_0 : \mu = \mu_0$ : If  $\sigma_Y$  is known, use:  $\frac{(\bar{y} - \mu_0)}{\sigma_Y / \sqrt{n}}$  and the standard Normal distribution; otherwise, use:  $\frac{(\bar{y} - \mu_0)}{s_Y / \sqrt{n}}$  and the  $t_{n-1}$  - distribution.

**Hypothesis Tests for the Difference in Means for Independent Samples:**

To test against  $H_0 : \mu_Y = \mu_X$ : If  $\sigma_Y$  and  $\sigma_X$  are known, use  $\frac{\bar{y} - \bar{x}}{\sqrt{\frac{\sigma_Y^2}{n} + \frac{\sigma_X^2}{m}}}$  as a test statistic.

If  $\sigma_Y$  and  $\sigma_X$  are unknown but assumed equal, use  $\frac{\bar{y} - \bar{x}}{s_P \sqrt{\frac{1}{n} + \frac{1}{m}}}$  as a test statistic.

**Single Factorial Models (analysed through ANOVA):**

For factor levels  $i=1, \dots, k$ ,  $Y_{i,j} = \mu_i + \varepsilon_{i,j}$ , with  $\varepsilon_{i,j} \sim N(0, \sigma^2)$ .

**Simple Linear Regression (estimated through least squares):**

$$Y_j = \beta_0 + \beta_1 x_j + \varepsilon_j, \text{ with } \varepsilon_j \sim N(0, \sigma^2).$$

### Standard Normal Cumulative Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002	0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003	0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0006	.0005	.0005	0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007	0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010	0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014	0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019	0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026	0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036	0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048	0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064	1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084	1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110	1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143	1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183	1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233	1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
-1.8	.0359	.0351	.0344	.0338	.0329	.0322	.0314	.0307	.0301	.0294	1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367	1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455	1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559	1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681	2.0	.9773	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823	2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985	2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170	2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379	2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611	2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867	2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
-0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148	2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451	2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776	2.9	.9981	.9982	.9983	.9983	.9984	.9984	.9985	.9985	.9986	.9986
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121	3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483	3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859	3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247	3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641	3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

This table was generated using the "CDF" command in Minitab.

**t-Distribution Quantiles**

$\nu$	Q(.9)	Q(.95)	Q(.975)	Q(.99)	Q(.995)	Q(.999)	Q(.9995)
1	3.078	6.314	12.706	31.821	63.657	318.317	636.607
2	1.886	2.920	4.303	6.965	9.925	22.327	31.598
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.849
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
$\infty$	1.282	1.645	1.960	2.326	2.576	3.090	3.291

This table was generated using the "INVCDF" command in Minitab.

**END OF EXAMINATION**